DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS SESSION D12

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D1

THE FRACTIONAL SUSCEPTIBILITY FUNCTION FOR THE QUADRATIC FAMILY

Viviane Baladi
CNRS & Sorbonne Université, France
Joint work with **Daniel Smania**.

For t in a set Ω of positive measure, maps in the quadratic family $f_t(x) = t - x^2$ admit an SRB measure μ_t. On the one hand, the dependence of μ_t on t has been shown [1] to be no better than 1/2 Hölder, on a subset of Ω, for t_0 a suitable Misiurewicz-Thurston parameter. On the other hand, the susceptibility function $\Psi_t(z)$, whose value at $z = 1$ is a candidate for the derivative of μ_t with respect to t, has been shown [2] to admit a holomorphic extension at $z = 1$ for $t = t_0$. Our goal is to resolve this paradox. For this, we introduce and study a fractional susceptibility function.

References

[1] V. Baladi, M. Benedicks, and D. Schnellmann, *Whitney Hölder continuity of the SRB measure for transversal families of smooth unimodal maps*, Invent. Math. **201** (2015), 773-844.

[2] Y. Jiang, D. Ruelle, *Analyticity of the susceptibility function for unimodal Markovian maps of the interval*, Nonlinearity **18** (2005), 2447-2453.

UNIQUE ERGODICITY FOR FOLIATIONS ON COMPACT KAEHLER SURFACES

Tien-Cuong Dinh
National University of Singapore, Singapore
Joint work with **Viet-Anh Nguyen and Nessim Sibony**.

Let F be a holomorphic foliation by Riemann surfaces on a compact Kaehler surface. Assume it is generic in the sense that all the singularities are hyperbolic and that the foliation admits no directed positive closed $(1,1)$-current, or equivalently, no invariant measure. Then there exists a unique (up to a multiplicative constant) positive ddc-closed $(1,1)$-current directed by F, or equivalently, a unique harmonic measure. This is a very strong ergodic property showing that all leaves of F have the same asymptotic behavior. Our proof uses an extension of the theory of densities to a new class of currents. A complete description of the cone of directed positive ddc-closed $(1,1)$-currents (i.e. harmonic measures) is also given when F admits directed positive closed currents (i.e. invariant measures).

MEASURE RIGIDITY FOR HIGHER RANK DIAGONALIZABLE ACTIONS

Manfred Einsiedler
ETH Zürich, Switzerland
Joint work with **Elon Lindenstrauss**.

We review old and recent measure rigidity results for higher rank diagonalizable actions on homogeneous spaces and contrast these results with the rank one and unipotent case. After this we consider higher rank actions on irreducible arithmetic quotients of $\mathrm{SL}_2(\mathbb{R})^k$ for $k \geq 2$. If the quotient is compact, positive entropy of an ergodic invariant measure μ implies algebraicity of μ with semisimple stabiliser. For non-compact quotients there are more possibilities. The main novelty here is that the acting group does not have to be maximal or in a special position. The main new idea is to use a quantitative recurrence phenomenon to transport positivity of entropy for one acting element to another.

EQUIDISTRIBUTION FOR COMMUTING MAPS

Michael Hochman
Hebrew University of Jerusalem, Israel

In two classical papers circa 1960, J. Cassels and W. Schmidt proved that a.e. numbers in the ternary Cantor set (with respect to Cantor-Lebesgue measure) eqidistributes for Lebesgue measure under the map $Tx = bx \bmod 1$, whenever b is an integer that is not a power of 3. This phenomenon has since been established in much greater generality on the interval, e.g. Host's theorem, according to which one can replace Cantor-Lebesgue measure by any $\times 3$-ergodic measure of positive entropy, provided $\gcd(3, b) = 1$. In this talk I will describe a new and heuristically simple proof of such results, and then discuss how it can be extended to give new results in multi-dimensional settings.

ON DYNAMICAL SPECTRAL RIGIDITY OF PLANAR DOMAINS

Vadim Kaloshin
University of Maryland, College Park, USA

Consider a convex domain on the plane and the associated billiard inside. The length spectrum is the closure of the union of perimeters of all period orbits. The length spectrum is closely related to the Laplace spectrum, through so-called the wave trace. The well-known question popularized by M. Kac: "Can you hear the shape of a drum?" asks if the Laplace spectrum determines a domain up to isometry. We call a domain dynamically spectrally rigid (DSR) if any smooth deformation preserving the length spectrum is an isometry. During the talk I will discuss recent results on DSR of convex planar domains.

ON THE DIVERGENCE OF BIRKHOFF NORMAL FORMS

Raphaël Krikorian
CNRS & Université de Cergy-Pontoise, France

A real analytic hamiltonian or a real analytic exact symplectic diffeomorphism admitting a non resonant elliptic fixed point is always formally conjugated to a formal integrable system, its Birkhoff Normal Form (BNF). Siegel proved in 1954 that the formal conjugation reducing a hamiltonian to its BNF is in general divergent and Hakan Eliasson has asked whether the BNF itself could be divergent. Perez-Marco proved in 2001 that for any fixed non resonant frequency vector the following dichotomy holds: either any real analytic hamiltonian system admitting this frequency vector at the origin has a convergent BNF or for a prevalent set of hamiltonians admitting this frequency vector the BNF generically diverges. It is possible to exhibit examples of hamiltonian systems with diverging BNF (X. Gong 2012 or the recent examples of B. Fayad in 4 degrees of freedom). The aim of this talk is to give a complete answer to the question of the divergence of the BNF (in the setting of exact symplectic diffeomorphisms): for any non resonant frequency vector, the BNF of a real analytic exact symplectic diffeomorphism admitting this frequency vector at the origin, is in general divergent. This theorem is the consequence of the remarkable fact that the convergence of the formal object that is the BNF has dynamical consequences, in particular an abnormal abundance of invariant tori.

KINETIC THEORY FOR THE LOW-DENSITY LORENTZ GAS

Jens Marklof
University of Bristol, UK
Joint work with **Andreas Strombergsson**.

The Lorentz gas is one of the simplest and most widely-studied models for particle transport in matter. It describes a cloud of non-interacting gas particles in an infinitely extended array of identical spherical scatterers, whose radii are small compared to their mean separation. The model was introduced by Lorentz in 1905 who, following the pioneering ideas of Maxwell

and Boltzmann, postulated that its macroscopic transport properties should be governed by a linear Boltzmann equation. A rigorous derivation of the linear Boltzmann equation from the underlying particle dynamics was given, for random scatterer configurations, in three seminal papers by Gallavotti, Spohn and Boldrighini-Bunimovich-Sinai. The objective of this lecture is to develop an approach for a large class of deterministic scatterer configurations, including various types of quasicrystals. We prove the convergence of the particle dynamics to transport processes that are in general (depending on the scatterer configuration) not described by the linear Boltzmann equation. This was previously understood only in the case of the periodic Lorentz gas through work of Caglioti-Golse and Marklof-Strombergsson. Our results extend beyond the classical Lorentz gas with hard sphere scatterers, and in particular hold for general classes of spherically symmetric finite-range potentials. We employ a rescaling technique that randomises the point configuration given by the scatterers' centers. The limiting transport process is then expressed in terms of a point process that arises as the limit of the randomised point configuration under a certain volume-preserving one-parameter linear group action.

INFLECTION POINTS FOR LYAPUNOV SPECTRA

Mark Pollicott
University of Warwick, UK
Joint work with **Oliver Jenkinson and Polina Vytnova**.

The Lyapunov spectra for a dynamical system describes the size (Hausdorff dimension) of the set of points which have a given Lyapunov exponent. H. Weiss conjectured that the associated graph is convex, but Iommi and Kiwi constructed a simple counter example. We explore this problem further, constructing examples with any given number of points of inflection.

MANDELBROT SET SEEN BY HARMONIC MEASURE: THE SIMILARITY MAP

Grzegorz Świątek
Warsaw University of Technology, Poland
Joint work with **Jacek Graczyk**.

We study conformal quantities at generic parameters with respect to the harmonic measure on the boundary of the connectedness loci $\mathcal{M}_d$ for unicritical polynomials $f_c(z) = z^d + c$. It is known that these parameters are structurally unstable and have stochastic dynamics. In [3] it was shown that for c from a set of full harmonic measure in $\partial\mathcal{M}_d$ there exists a quasi-conformal similarity map Υ_c between phase and parameter spaces which is conformal at c. In a recent work [2] we prove $C^{1+\frac{\alpha}{d}-\epsilon}$-conformality, $\alpha = \mathrm{HD}(\mathcal{J}_c)$, of $\Upsilon_c(z) : \mathbb{C} \mapsto \mathbb{C}$ at typical $c \in \partial\mathcal{M}_d$ and establish that globally quasiconformal similarity maps $\Upsilon_c(z)$, $c \in \partial\mathcal{M}_d$, are C^1-conformal along external rays landing at c in $\mathbb{C} \setminus \mathcal{J}_c$ mapping onto the corresponding rays of $\mathcal{M}_d$. This conformal equivalence leads to a proof that the z-derivative of the similarity map $\Upsilon_c(z)$ at typical $c \in \partial\mathcal{M}_d$ is equal to $1/\mathcal{T}'(c)$, where

$$\mathcal{T}(c) = \sum_{n=0}^{\infty} \left(D_z \left[f_c^n(z)\right]_{z=c}\right)^{-1}$$

is the transversality function previously studied by Benedicks-Carleson and Levin, see [1, 4]. There are additonal geometric consequences of these results. A typical external radius of the connectedness locus is contained in an asymptotically very nearly linear twisted angle, but nevertheless passes through infinitely many increasingly narrow straits.

References

[1] M. Benedicks, L. Carleson, *On iterations of* $1 - ax^2$ *on* $(-1, 1)$, Ann. of Math. **122** (1985), 1-25.

[2] J. Graczyk, G. Świątek, *Analytic structures and harmonic measure at bifurcation locus*, arXiv 1904.09434 (2019).

[3] J. Graczyk, G. Świątek, *Fine structure of connectedness loci*, Math. Ann. **369** (2017), 49-108.

[4] G. Levin, *An analytical approach to the Fatou conjecture*, Fund. Math. **171** (2002), 177-196.

TALKS OF SESSION D12

WANDERING DOMAINS ARISING WITH FROM LAVAURS MAPS WITH SIEGEL DISKS

Matthieu Astorg
Université d'Orléans, France

A famous theorem of Sullivan asserts that polynomials in one complex variables have no wandering Fatou components. On the other hand, in a joint work with Buff, Dujardin, Peters and Raissy, we constructed the first examples of polynomial maps in two complex variables having such components. The construction relies on parabolic implosion, and involves the dynamics of non-autonomous perturbations of a Lavaurs map with an attracting fixed point. In this talk, we will present a more recent work with Boc-Thaler and Peters, in which we classify the local dynamics in the case where the Lavaurs map has a Siegel fixed point. In particular, we prove that wandering domains may also arise in that setting. Time permitting, we will introduce the notion of parabolic curves and how their existence simplifies the proof

SLOW ESCAPING POINTS FOR TRANSCENDENTAL MAPS

Krzysztof Barański
University of Warsaw, Poland
Joint work with **Bogusława Karpińska**.

Let $f\colon \mathbb{C} \to \mathbb{C}$ be a transcendental entire map. The set

$$I(f) = \{z \in \mathbb{C} : f^n(z) \to \infty\}$$

is called the *escaping set* of f. In relation with the papers [1, 2], we study the dimension of the sets of points in $I(f)$ which escape to infinity in a given rate.

References

[1] W. Bergweiler, J. Peter, *Escape rate and Hausdorff measure for entire functions*, Math. Z. **274** (2013), 551-572.

[2] D.J. Sixsmith, *Dimensions of slowly escaping sets and annular itineraries for exponential functions*, Ergodic Theory Dynam. Systems **36** (2016), 2273-2292.

LATTES MAPS AND THE HAUSDORFF DIMENSION OF THE BIFURCATION LOCUS

Fabrizio Bianchi
CNRS & Université de Lille, France
Joint work with **François Berteloot**.

Given a holomorphic family of endomorphisms of $P^k(C)$, the bifurcation locus is the set of parameters of instability for the Julia set, the support of the measure of maximal entropy.

This locus coincides with the non-harmonicity locus of the Lyapunov function, the sum of the Lyapunov exponents. Lattes maps can be characterised as the minima of the Lyapunov function, and thus lie in the bifurcation locus. We will prove that near isolated Lattes maps the Hausdorff dimension of the bifurcation locus is maximal in any direction.

LOCATION OF SIEGEL CAPTURE POLYNOMIALS IN PARAMETER SPACES OF CUBIC POLYNOMIALS

Alexander Blokh
University of Alabama at Birmingham, USA
Joint work with **Arnaud Chéritat, Lex Oversteegen, and Vladlen Timorin**.

Consider cubic polynomials with a Siegel disk containing an eventual image of a critical point and call them *IS-capture polynomials* ("IS" stands for Invariant Siegel). We study the location of IS-capture polynomials in the parameter space of all cubic polynomials and show that any IS-capture polynomial belongs to the boundary of a unique bounded hyperbolic component determined by the rational lamination of the map.

CONJUGACY CLASSES OF REAL ANALYTIC MAPPINGS

Trevor Clark
Imperial College London, UK
Joint work with **Sebastian van Strien**.

I will discuss recent results on the manifold structure of the topological conjugacy classes of real-analytic mappings. These results are based on the construction of a "pruned polynomial-like

mapping" associated to a real mapping. This gives us an "external structure" for a real-analytic mapping.

DYNAMICS OF UNIFORMLY HYPERBOLIC HÉNON MAPS

Romain Dujardin
Sorbonne Université, France
Joint work with **Eric Bedford and Misha Lyubich**.

In this talk I will review some new recent results on the dynamics of uniformly hyperbolic Hénon maps. Topics covered will include the $J = J^*$ problem, some new geometric and topological criteria for hyperbolicity, and the topological structure of the Julia set for hyperbolic maps.

DYNAMICS OF FIBERED ENDOMORPHISMS OF $\mathbb{CP}(2)$

Christophe Dupont
Université de Rennes 1, France
Joint work with **Johan Taflin**.

The talk concerns the endomorphisms of $\mathbb{CP}(2)$ preserving a pencil of lines, those maps generalize the polynomial skew products of $\mathbb{C}^2$ studied by Jonsson. We show that the equilibrium measure of those endomorphisms decomposes (Fubini's formula relative to the invariant pencil) and we study its Lyapunov exponents. One of them is equal to the exponent of the rational map acting on the pencil. We provide for the other one a formula involving a relative Green function and the critical set. In particular, that exponent is larger than the logarithm of the degree of the endomorphism. This is a joint work with Johan Taflin.

WANDERING DOMAINS IN TRANSCENDENTAL FUNCTIONS

Núria Fagella
University of Barcelona, Spain

Wandering domains are Fatou components that only exist in the transcendental setting. Although important progress has taken place in the past few years, there are still many open questions. In this talk I will review the state of the art on the existence and classification of wandering domains, and their relation with the singularities of the inverse map. I shall present some recent results on these topics.

ON RENORMALIZATION OF CRITICAL CIRCLE MAPS WITH NONINTEGER EXPONENTS

Igors Gorbovickis
Jacobs University, Germany

We discuss some results and ongoing developments in the study of renormalization of critical circle maps with non-integer critical exponents sufficiently close to odd integers.

References

[1] I. Gorbovickis, M. Yampolsky, *Rigidity, universality,and hyperbolicity of renormalization for critical circle maps with non-integer exponents*, to appear in Ergodic Theory Dynam. Systems.

[2] I. Gorbovickis, M. Yampolsky, *Renormalization for unimodal maps with non-integer exponents*, Arnold Math. Journal **4(2)** (2018), 179-191.

UNIVALENT WANDERING DOMAINS IN THE EREMENKO-LYUBICH CLASS

Xavier Jarque
Universitat de Barcelona & IMUB, Catalonia
Joint work with **Núria Fagella and Kirill Lazebnik**.

We use the Folding Theorem of [1] to construct an entire function f in class $\mathcal{B}$ and a wandering domain U of f such that f restricted to $f^n(U)$ is univalent, for all $n \geq 0$. The components of the wandering orbit are bounded and surrounded by the postcritical set.

References

[1] C. Bishop, *Constructing entire functions by quasiconformal folding*, Acta Mathematica **214(1)** (2015), 1-60.

NON-UNIFORM HYPERBOLICITY IN POLYNOMIAL SKEW PRODUCTS

Zhuchao Ji
Sorbonne Université, France

The dynamics of Topological Collet-Eckmann rational maps on Riemann sphere are well understood, due to the work of Przytycki, Rivera-Letelier and Smirnov [1, 2]. In this talk we study the dynamics of polynomial skew products of $\mathbb{C}^2$. Let f be a polynomial skew products with an attracting invariant line L, such that f restricted on L satisfies Topological Collet-Eckmann condition and a Weak Regularity condition. We show that the the Fatou set of f in the basin of L equals to the union of the basins of attracting cycles, and the Julia set of f in the basin of L has Lebesgue measure zero. As a consequence there are no wandering Fatou components in the basin of L (we remark that for some polynomial skew products with a parabolic invariant line L, there can exist a wandering Fatou component in the basin of L [3, 4]).

References

[1] F. Przytycki, J. Rivera-Letelier and S. Smirnov, *Equivalence and topological invariance of conditions for non-uniform hyperbolicity in the iteration of rational maps*, Inventiones mathematicae **151** (2003), 29–63.

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[3] M. Astorg, X. Buff, R. Dujardin, H. Peters and J. Raissy, *A two-dimensional polynomial mapping with a wandering Fatou component*, Annals of mathematics **184** (2016), 263– 313.

[4] M. Astorg, L. Boc-Thaler and H. Peters, *Wandering domains arising from Lavaurs maps with Siegel disks*, arXiv preprint arXiv:1907.04140 (2019).

[5] Z. Ji, *Non-uniform hyperbolicity in polynomial skew products*, In preparation.

UNIVALENT POLYNOMIALS AND HUBBARD TREES

Kirill Lazebnik
California Institute of Technology, USA
Joint work with **Nikolai G. Makarov and Sabyasachi Mukherjee**.

We study the space of "external polynomials"

$$\Sigma_d^* := \left\{ f(z) = z + \frac{a_1}{z} + \cdots + \frac{a_d}{z^d} : a_d = -\frac{1}{d} \text{ and } f|_{\hat{\mathbb{C}}\setminus\overline{\mathbb{D}}} \text{ is conformal} \right\}.$$

It is proven that a simple class of combinatorial objects (*bi-angled trees*) classify those $f \in \Sigma_d^*$ with the property that $f(\mathbb{T})$ has the maximal number $d-2$ of double points. We discuss a surprising connection with the class of anti-holomorphic polynomials of degree d with $d-1$ distinct, fixed critical points and their associated Hubbard trees.

References

[1] Lazebnik, Kirill, Makarov, Nikolai, Mukherjee, Sabyasachi, *Univalent Polynomials and Hubbard Trees*, arXiv, 2019.

FINGERS IN THE PARAMETER SPACE OF THE COMPLEX STANDARD FAMILY

David Martí-Pete
Polish Academy of Sciences, Poland
Joint work with **Mitsuhiro Shishikura**.

We investigate the parameter space of the complex standard family

$$F_{\alpha,\beta}(z) = z + \alpha + \beta \sin z,$$

where the parameter $0 < \beta \ll 1$ is considered to be fixed and the bifurcation is studied with respect to the parameter $\alpha \in \mathbb{C}$. This two parameter family of entire functions are lifts of holomorphic self-maps of $\mathbb{C}^*$ that arise as the complexification of the Arnol'd standard family of circle maps. In the real axis of the α-parameter plane one can observe the so-called Arnold tongues, given by the real parameters (α, β) such that $F_{\alpha,\beta}$ has a constant rotation number. Their complex extension contain some finger-like structures which were observed for the first time by Fagella in her PhD thesis [1] that increase in number as $\beta \to 0$. We study the qualitative and quantitative aspects of such fingers via parabolic bifurcation. In particular, we show that for every $0 < \beta \ll 1$ the number of fingers is finite and give an estimate of this quantity as $\beta \to 0$. This is a very general capture phenomenon that appears in the parameter spaces of many families of holomorphic functions with more than one critical point.

References

[1] N. Fagella, *Dynamics of the complex standard family*, J. Math. Anal. Appl. **229(1)** (1999), 1-31.

CRITICAL PORTRAITS, SIBLING PORTRAITS, THE CENTRAL STRIP, AND NEVER CLOSE SIDES OF POLYGONS IN LAMINATIONS

John Mayer
University of Alabama at Birmingham, USA

Laminations of the unit disk were introduced by William Thurston as a topological/combinatorial vehicle for understanding the (connected) Julia sets of polynomials, and, in particular, the parameter space of quadratic polynomials. Though the problem that Thurston was interested in has not been solved, the local connectedness of the Mandelbrot set (the analytic parameter space of quadratic polynomials), his excursion into laminations eventually gave birth to laminations as a way of understanding higher degree polynomials and their corresponding laminations. Much work has been done for cubic polynomials and their parameter spaces (analytic and laminational). In this talk we will decribe some work in progress on understanding phenomena that can occur with laminations, and consequently with Julia sets (maybe), of higher degree, $d \geq 3$. In particular, we are interested in laminational phenomena that cannot occur for $d = 2$, but can occur for $d = 3$, cannot occur for $d \leq 3$, but can occur for $d = 4$, and so on. The topics mentioned in the title are on the route of discovery.

References

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[3] D. Childers, *Wandering polygons and recurrent critical leaves*, Ergodic Theory Dynam. Systems **27(1)** (2007), 87-107.

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[5] J. Mayer, L. Mernik, *Central Strip Portraits*, preprint (2017).

THE ITERATED MINIMUM MODULUS AND EREMENKO'S CONJECTURE

Daniel Nicks
University of Nottingham, UK
Joint work with **Phil Rippon and Gwyneth Stallard**.

Eremenko has conjectured that for any transcendental entire function f, the escaping set $I(f) := \{z : f^n(z) \to \infty \text{ as } n \to \infty\}$ is connected. This talk will focus on real entire functions of finite order with only real zeroes. We show that Eremenko's conjecture holds for such a function f (and in fact $I(f)$ has a "spider's web" structure) if there exists $r > 0$ such that the iterated minimum modulus $m^n(r) \to \infty$ as $n \to \infty$. Here $m(r) := \min_{|z|=r} |f(z)|$. We will briefly discuss examples of families of functions for which this minimum modulus condition does, and does not, hold.

SLICES OF THE PARAMETER SPACE OF CUBIC POLYNOMIALS

Lex Oversteegen
University of Alabama at Birmingham, USA
Joint work with **Alexander Blokh and Vladlen Timorin**.

In this talk we consider slices of the parameter space of cubic polynomials, up to affine conjugacy, given by a fixed value of the multiplier at a non-repelling fixed point. In particular, we study the location of the main cubioid in this parameter space. The main cubioid is the set of affine conjugacy classes of complex cubic polynomials that have certain dynamical properties generalizing those of polynomials $z^2 + c$ for c in the filled main cardioid.

A DYNAMICAL RUNGE EMBEDDING OF $\mathbb{C} \times \mathbb{C}^*$ IN $\mathbb{C}^2$

Jasmin Raissy
Université Paul Sabatier, France
Joint work with **Filippo Bracci and Berit Stensønes**.

In this talk, I will present the construction of a family of automorphisms of $\mathbb{C}^2$ having an invariant, non-recurrent Fatou component biholomorphic to $\mathbb{C} \times \mathbb{C}^*$ and which is attracting, in the sense that all the orbits converge to a fixed point on the boundary of the component. Such component is obtained by globalizing, thanks to a result of Forstneric, a local construction, which allows to create a global basin of attraction for an automorphism, and a Fatou coordinate on it. Such Fatou coordinate is a fiber bundle map on $\mathbb{C}$, whose fiber is $\mathbb{C}^*$, forcing the global basin to be biholomorphic to $\mathbb{C} \times \mathbb{C}^*$. The most subtle point is to show that such a basin is indeed a Fatou component. This is done exploiting Pöschel's results about existence of local Siegel discs and suitable estimates for the Kobayashi distance. This construction gives an example of a Runge embedding of $\mathbb{C} \times \mathbb{C}^*$ in $\mathbb{C}^2$, since attracting Fatou components are Runge domains.

References

[1] F. Bracci, J. Raissy, B. Stensønes, *Automorphisms of $\mathbb{C}^k$ with an invariant non-recurrent attracting Fatou component biholomorphic to $\mathbb{C} \times (\mathbb{C}^*)^{k-1}$*, to appear in Journal of the European Mathematical Society, www.ems-ph.org/journals/forthcoming.php?jrn=jems, arXiv:1703.08423.

CONSTRUCTING BOUNDED SIMPLY CONNECTED WANDERING DOMAINS WITH PRESCRIBED DYNAMICS

Philip Rippon
Open University, UK
Joint work with **Anna Miriam Benini, Vasiliki Evdoridou, Nuria Fagella, and**

Gwyneth Stallard.

We give a new general technique for constructing transcendental entire functions with bounded simply connected wandering domains, which allows us to prescribe the type of long term dynamics *within* the wandering domains. In some cases we can show that the wandering domains have Jordan curve boundaries.

THE RATIONAL RIGIDITY PRINCIPLE: TOWARDS RIGIDITY OF RATIONAL MAPS

Dierk Schleicher
Aix Marseille Université, France
Joint work with **Kostiantyn Drach**.

Rigidity is one of the key goals in holomorphic dynamics; one way to phrase it is to say that any two maps can be distinguished in combinatorial terms. There are deep results and remarkable progress especially about polynomial maps. Much less is known about non-polynomial rational maps. We present recent progress, in joint work with Kostiantyn Drach, in establishing rigidity of a large class of rational maps, in particular Newton maps of polynomials. Similarly, we prove local connectivity of the corresponding Julia sets.

PACMAN RENORMALIZATION

Nikita Selinger
University of Alabama at Birmingham, USA
Joint work with **Dima Dudko and Misha Lyubich**.

In a joint work with Misha Lyubich and Dima Dudko, we develop a theory of Pacman Renormalization inspired by earlier surgery construction by Branner and Douady. We show the

hyperbolicity of periodic points of this renormalization with one-dimensional unstable manifold. This yield multiple consequences such as scaling law for centers of hyperbolic components attached to the main cardioid of the Mandelbrot set and local stability of certain Siegel discs.

CLASSIFYING SIMPLY CONNECTED WANDERING DOMAINS

Gwyneth Stallard
Open University, UK
Joint work with **Anna Miriam Benini, Vasiliki Evdoridou, Nuria Fagella, and Phil Rippon**.

For rational functions, a classification of periodic Fatou components, with a detailed description of the dynamical behaviour inside each of the four possible types, was given around 100 years ago. For transcendental entire functions, there is an additional class known as Baker domains that are now well understood, and many examples of wandering domains, which cannot occur for rational functions. Although there is now a detailed description of the dynamical behaviour inside multiply connected wandering domains, there has been no systematic study of simply connected wandering domains. We show that there is in fact a wealth of possibilities for such domains and give a new classification into nine different types in terms of the hyperbolic distance between iterates and by whether orbits approach the boundaries of the domains. We give a new general technique for constructing bounded simply connected wandering domains which can be used to show that all nine types are realisable.

REGULARITY OF ATTRACTING CURRENTS

Johan Taflin
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To each attractor of an endomorphism f of $\mathbb{CP}^k$ it is possible to associate an analytic object called an *attracting current*. It can be used to obtain several information on the attractor and in this talk, I will explain how a weak form of regularity of this current is related to the dynamics of f on the attractor.

INVARIANT SPANNING TREES FOR QUADRATIC RATIONAL MAPS

Vladlen Timorin
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Joint work with **Anastasia Shepelevtseva**.

A theorem of W. Thurston (sometimes called the fundamental theorem of complex dynamics) opens a door for algebraic, topological and combinatorial methods into dynamics of rational maps on the Riemann sphere. We study Thurston equivalence classes of quadratic post-critically finite branched coverings. For these maps, we introduce and study *invariant spanning trees*. We give a computational procedure for searching for invariant spanning trees. This procedure uses bisets over the fundamental group of a punctured sphere. We also introduce a new combinatorial invariant of Thurston classes - the ivy graph representing the pullback relation on (isotopy classes of) spanning trees.

NON-AUTONOMOUS PARABOLIC BIFURCATION

Liz Vivas
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Let $f(z) = z + z^2 + O(z^3)$ and $f_\epsilon(z) = f(z) + \epsilon^2$. A classical result in parabolic bifurcation [1, 2] in one complex variable is the following: if $N - \frac{\pi}{\epsilon} \to 0$ we obtain $(f_\epsilon)^N \to \mathcal{L}_f$, where $\mathcal{L}_f$ is the

Lavaurs map of f. In this paper we study a *non-autonomous* parabolic bifurcation. We focus on the case of $f_0(z) = \frac{z}{1-z}$. Given a sequence $\{\epsilon_i\}_{1\leq i\leq N}$, we denote $f_n(z) = f_0(z) + \epsilon_n^2$. We give sufficient and necessary conditions on the sequence $\{\epsilon_i\}$ that imply that $f_N \circ \dots f_1 \to \mathrm{Id}$ (the Lavaurs map of f_0). We apply our results to prove parabolic bifurcation phenomenon in two dimensions for some class of maps.

References

[1] A.Douady, *Does a Julia set depend continuously on the polynomial?*, Proc. Sympos. Appl. Math. **49** (1994), 91-138.

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RECURRENCE OF ONE-SIDED SEQUENCES UNDER SHIFT

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Consider the shift map acting on the space of one-sided sequences. Under this dynamics, a sequence exhibits one of three types of recurrence: non-recurrence, reluctant recurrence, or persistent recurrence. However, for a given arbitrary sequence, it can be difficult to determine which of these three possibilities will occur. To solve this problem, we introduce an algebraic structure on sequences called filtration that enables us to count recurrences efficiently. This then leads to the characterization of the persistent recurrence property as a kind of infinite renormalizability of the shift map.