DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS SESSION D14

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D1

THE FRACTIONAL SUSCEPTIBILITY FUNCTION FOR THE QUADRATIC FAMILY

Viviane Baladi
CNRS & Sorbonne Université, France
Joint work with **Daniel Smania**.

For t in a set Ω of positive measure, maps in the quadratic family $f_t(x) = t - x^2$ admit an SRB measure μ_t. On the one hand, the dependence of μ_t on t has been shown [1] to be no better than 1/2 Hölder, on a subset of Ω, for t_0 a suitable Misiurewicz-Thurston parameter. On the other hand, the susceptibility function $\Psi_t(z)$, whose value at $z = 1$ is a candidate for the derivative of μ_t with respect to t, has been shown [2] to admit a holomorphic extension at $z = 1$ for $t = t_0$. Our goal is to resolve this paradox. For this, we introduce and study a fractional susceptibility function.

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UNIQUE ERGODICITY FOR FOLIATIONS ON COMPACT KAEHLER SURFACES

Tien-Cuong Dinh
National University of Singapore, Singapore
Joint work with **Viet-Anh Nguyen and Nessim Sibony**.

Let F be a holomorphic foliation by Riemann surfaces on a compact Kaehler surface. Assume it is generic in the sense that all the singularities are hyperbolic and that the foliation admits no directed positive closed $(1,1)$-current, or equivalently, no invariant measure. Then there exists a unique (up to a multiplicative constant) positive ddc-closed $(1,1)$-current directed by F, or equivalently, a unique harmonic measure. This is a very strong ergodic property showing that all leaves of F have the same asymptotic behavior. Our proof uses an extension of the theory of densities to a new class of currents. A complete description of the cone of directed positive ddc-closed $(1,1)$-currents (i.e. harmonic measures) is also given when F admits directed positive closed currents (i.e. invariant measures).

MEASURE RIGIDITY FOR HIGHER RANK DIAGONALIZABLE ACTIONS

Manfred Einsiedler
ETH Zürich, Switzerland
Joint work with **Elon Lindenstrauss**.

We review old and recent measure rigidity results for higher rank diagonalizable actions on homogeneous spaces and contrast these results with the rank one and unipotent case. After this we consider higher rank actions on irreducible arithmetic quotients of $\mathrm{SL}_2(\mathbb{R})^k$ for $k \geq 2$. If the quotient is compact, positive entropy of an ergodic invariant measure μ implies algebraicity of μ with semisimple stabiliser. For non-compact quotients there are more possibilities. The main novelty here is that the acting group does not have to be maximal or in a special position. The main new idea is to use a quantitative recurrence phenomenon to transport positivity of entropy for one acting element to another.

EQUIDISTRIBUTION FOR COMMUTING MAPS

Michael Hochman
Hebrew University of Jerusalem, Israel

In two classical papers circa 1960, J. Cassels and W. Schmidt proved that a.e. numbers in the ternary Cantor set (with respect to Cantor-Lebesgue measure) eqidistributes for Lebesgue measure under the map $Tx = bx \bmod 1$, whenever b is an integer that is not a power of 3. This phenomenon has since been established in much greater generality on the interval, e.g. Host's theorem, according to which one can replace Cantor-Lebesgue measure by any $\times 3$-ergodic measure of positive entropy, provided $\gcd(3, b) = 1$. In this talk I will describe a new and heuristically simple proof of such results, and then discuss how it can be extended to give new results in multi-dimensional settings.

ON DYNAMICAL SPECTRAL RIGIDITY OF PLANAR DOMAINS

Vadim Kaloshin
University of Maryland, College Park, USA

Consider a convex domain on the plane and the associated billiard inside. The length spectrum is the closure of the union of perimeters of all period orbits. The length spectrum is closely related to the Laplace spectrum, through so-called the wave trace. The well-known question popularized by M. Kac: "Can you hear the shape of a drum?" asks if the Laplace spectrum determines a domain up to isometry. We call a domain dynamically spectrally rigid (DSR) if any smooth deformation preserving the length spectrum is an isometry. During the talk I will discuss recent results on DSR of convex planar domains.

ON THE DIVERGENCE OF BIRKHOFF NORMAL FORMS

Raphaël Krikorian
CNRS & Université de Cergy-Pontoise, France

A real analytic hamiltonian or a real analytic exact symplectic diffeomorphism admitting a non resonant elliptic fixed point is always formally conjugated to a formal integrable system, its Birkhoff Normal Form (BNF). Siegel proved in 1954 that the formal conjugation reducing a hamiltonian to its BNF is in general divergent and Hakan Eliasson has asked whether the BNF itself could be divergent. Perez-Marco proved in 2001 that for any fixed non resonant frequency vector the following dichotomy holds: either any real analytic hamiltonian system admitting this frequency vector at the origin has a convergent BNF or for a prevalent set of hamiltonians admitting this frequency vector the BNF generically diverges. It is possible to exhibit examples of hamiltonian systems with diverging BNF (X. Gong 2012 or the recent examples of B. Fayad in 4 degrees of freedom). The aim of this talk is to give a complete answer to the question of the divergence of the BNF (in the setting of exact symplectic diffeomorphisms): for any non resonant frequency vector, the BNF of a real analytic exact symplectic diffeomorphism admitting this frequency vector at the origin, is in general divergent. This theorem is the consequence of the remarkable fact that the convergence of the formal object that is the BNF has dynamical consequences, in particular an abnormal abundance of invariant tori.

KINETIC THEORY FOR THE LOW-DENSITY LORENTZ GAS

Jens Marklof
University of Bristol, UK
Joint work with **Andreas Strombergsson**.

The Lorentz gas is one of the simplest and most widely-studied models for particle transport in matter. It describes a cloud of non-interacting gas particles in an infinitely extended array of identical spherical scatterers, whose radii are small compared to their mean separation. The model was introduced by Lorentz in 1905 who, following the pioneering ideas of Maxwell

and Boltzmann, postulated that its macroscopic transport properties should be governed by a linear Boltzmann equation. A rigorous derivation of the linear Boltzmann equation from the underlying particle dynamics was given, for random scatterer configurations, in three seminal papers by Gallavotti, Spohn and Boldrighini-Bunimovich-Sinai. The objective of this lecture is to develop an approach for a large class of deterministic scatterer configurations, including various types of quasicrystals. We prove the convergence of the particle dynamics to transport processes that are in general (depending on the scatterer configuration) not described by the linear Boltzmann equation. This was previously understood only in the case of the periodic Lorentz gas through work of Caglioti-Golse and Marklof-Strombergsson. Our results extend beyond the classical Lorentz gas with hard sphere scatterers, and in particular hold for general classes of spherically symmetric finite-range potentials. We employ a rescaling technique that randomises the point configuration given by the scatterers' centers. The limiting transport process is then expressed in terms of a point process that arises as the limit of the randomised point configuration under a certain volume-preserving one-parameter linear group action.

INFLECTION POINTS FOR LYAPUNOV SPECTRA

Mark Pollicott
University of Warwick, UK
Joint work with **Oliver Jenkinson and Polina Vytnova**.

The Lyapunov spectra for a dynamical system describes the size (Hausdorff dimension) of the set of points which have a given Lyapunov exponent. H. Weiss conjectured that the associated graph is convex, but Iommi and Kiwi constructed a simple counter example. We explore this problem further, constructing examples with any given number of points of inflection.

MANDELBROT SET SEEN BY HARMONIC MEASURE: THE SIMILARITY MAP

Grzegorz Świątek
Warsaw University of Technology, Poland
Joint work with **Jacek Graczyk**.

We study conformal quantities at generic parameters with respect to the harmonic measure on the boundary of the connectedness loci $\mathcal{M}_d$ for unicritical polynomials $f_c(z) = z^d + c$. It is known that these parameters are structurally unstable and have stochastic dynamics. In [3] it was shown that for c from a set of full harmonic measure in $\partial\mathcal{M}_d$ there exists a quasi-conformal similarity map Υ_c between phase and parameter spaces which is conformal at c. In a recent work [2] we prove $C^{1+\frac{\alpha}{d}-\epsilon}$-conformality, $\alpha = \mathrm{HD}(\mathcal{J}_c)$, of $\Upsilon_c(z) : \mathbb{C} \mapsto \mathbb{C}$ at typical $c \in \partial\mathcal{M}_d$ and establish that globally quasiconformal similarity maps $\Upsilon_c(z)$, $c \in \partial\mathcal{M}_d$, are C^1-conformal along external rays landing at c in $\mathbb{C} \setminus \mathcal{J}_c$ mapping onto the corresponding rays of $\mathcal{M}_d$. This conformal equivalence leads to a proof that the z-derivative of the similarity map $\Upsilon_c(z)$ at typical $c \in \partial\mathcal{M}_d$ is equal to $1/\mathcal{T}'(c)$, where

$$\mathcal{T}(c) = \sum_{n=0}^{\infty} \left(D_z \left[f_c^n(z)\right]_{z=c}\right)^{-1}$$

is the transversality function previously studied by Benedicks-Carleson and Levin, see [1, 4]. There are additonal geometric consequences of these results. A typical external radius of the connectedness locus is contained in an asymptotically very nearly linear twisted angle, but nevertheless passes through infinitely many increasingly narrow straits.

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TALKS OF SESSION D14

ANALYTIC INVARIANT CURVES FOR ANALYTIC TWIST MAPS

Abed Bounemoura
CNRS & CEREMADE, Université Paris-Dauphine, France

Yoccoz proved that the Bruno arithmetical condition is optimal for the analytic linearization of a circle diffeomorphism close to a rotation. We will explain how to use this result to show that the same condition is optimal for the analytic preservation of quasi-periodic invariant curves for twist maps of the annulus, as well as new questions in this context.

INVARIANT TORI FOR A CLASS OF THERMOSTATED SYSTEMS

Leo Butler
University of Manitoba, Canada

A thermostated hamiltonian system is a model of a mechanical system immersed in a heat

bath at constant temperature. A fundamental question is whether the system reaches thermal equilibrium. Even for 1-degree-of-freedom hamiltonians this question is non-trivial, and an answer has only been known in the simplest case by work of Legoll, Luskin and Moeckel [4, 5].

Most of the literature on thermostats focuses on specific examples. I propose a mathematical definition of a thermostat that captures the content of these examples. Under some conditions, the properties of a thermostat lead to the existence of a thermostatic equilibrium when it is only weakly coupled with the hamiltonian. Under a modest additional hypothesis on the non-degeneracy of this equilibrium, one obtains the existence of invariant tori in a neighbourhood of the thermostatic equilibrium. The existence of such KAM tori frustrates "thermalization".

These conditions are verified for four well-known examples in the literature [1, 2, 3, 7, 8], when the hamiltonian is real-analytic and "well-behaved".

If time permits, I will discuss related results about variants including non-equilibrium thermostats, and multiple/recursive thermostats.

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CONTINUUM-WISE HYPERBOLICITY

Bernardo Carvalho
Federal University of Minas Gerais, Brazil & Friedrich-Schiller-Universität Jena, Germany
Joint work with **Alfonso Artigue, Welington Cordeiro, and José Vieitez**.

Hyperbolicity is one of the most important concepts in the theory of chaotic dynamical systems. Since the seminal works of Anosov [1] and Smale [5] it has been a main topic of research among many mathematicians. In hyperbolic systems, each tangent space splits into two invariant subspaces, the first being uniformly contracted, and the second uniformly expanded, by the action of the derivative map. The dynamics of such systems can be well described in both topological and statistical viewpoints, so many effort is being made to understand the dynamics *beyond uniform hyperbolicity* and many generalizations have been considered.

In this talk I will discuss one specific notion of hyperbolicity introduced in a joint work with A. Artigue, W. Cordeiro and J. Vieitez [3] called *continuum-wise hyperbolicity*. Examples of these systems are the Anosov diffeomorphisms [1], the topologically hyperbolic homeomorphisms and some pseudo-Anosov diffeomorphisms of the two-dimensional sphere. We discuss the dynamics of cw-hyperbolic homeomorphisms, proving that some dynamical properties that are present in the hyperbolic theory, such as the shadowing property and a spectral decomposition, are still present in cw-hyperbolic ones, while enlightening the differences and peculiarities of these systems, such as wilder local stable sets containing cantor sets and the existence of arbitrarily small horseshoes.

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CENTRALIZER CLASSIFICATION FOR SOME PARTIALLY HYPERBOLIC MAPS

Danijela Damjanović
Royal Institute of Technology (KTH), Sweden
Joint work with **Amie Wilkinson and Disheng Xu**.

In this talk I will survey recent advances on classification of centralizer for some conservative partially hyperbolic diffeomorphisms with one dimensional center foliation. I will describe how disintegration of volume along the center foliation [4] together with classification of higher rank partially hyperbolic abelian actions [1], [2] lead to classification results for centralizers [3]. I will also mention several conjectures concerning classification of centralizers.

References

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INFINITE LEBESGUE SPECTRUM FOR SURFACE FLOWS

Bassam Fayad
CNRS & Université Paris Diderot, France
Joint work with **Giovanni Forni and Adam Kanigowski**.

We study the spectral measures of conservative mixing flows on the two torus having one degenerate singularity. We show that, for a sufficiently strong singularity, the spectrum of these flows is typically Lebesgue with infinite multiplicity. For this, we use two main ingredients : 1) a proof of absolute continuity of the maximal spectral type for this class of non-uniformly stretching flows that have an irregular decay of correlations, 2) a geometric criterion that yields infinite Lebesgue multiplicity of the spectrum and that is well adapted to rapidly mixing flows, such as horocyclic flows on the unit tangent bundle M of a compact hyperbolic surface.

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EQUILIBRIUM STATES FOR CERTAIN PARTIALLY HYPERBOLIC ATTRACTORS

Todd Fisher
Brigham Young University, USA
Joint work with **Krerley Oliveira**.

I will discuss partially hyperbolic attractors introduced by Castro and Nascimento and show they have unique equilibrium states for natural classes of potentials. If the system is C^2, then there is a unique equilibrium states for the geometric potential and its 1-parameter family. The proofs follow by applying general techniques developed by Climenhaga and Thompson. This is joint work with Krerley Oliveira.

DYNAMICAL COUNTEREXAMPLES FOR THE USUAL INTERPRETATION OF THE EXTREMAL INDEX

Ana Cristina Moreira Freitas
University of Porto, Portugal

We consider stochastic processes arising from dynamical systems by evaluating an observable function along the orbits of the system. The existence of an Extremal Index less than 1 is associated to the occurrence of periodic phenomena, which is responsible for the appearance of clusters of exceedances. The Extremal Index usually coincides with the reciprocal of the mean of the limiting cluster size distribution. We build dynamically generated stochastic processes with an Extremal Index for which that relation does not hold. The mechanism used to build such counterexamples is based on considering observable functions maximised at at least two points of the phase space, where one of them is an indifferent periodic point.

RARE EVENTS FOR FRACTAL LANDSCAPES

Jorge Milhazes Freitas
University of Porto, Portugal

We consider the existence of limiting laws of rare events corresponding to the entrance of the orbits on certain target sets in the phase space. The limiting laws are obtained when the target sets shrink to fractal sets of zero Lebesgue measure. We consider both the presence and absence of clustering, which is detected by the Extremal Index, which turns out to be very useful to identify the compatibility between the dynamics and the fractal geometrical structure.

BILLIARDS INSIDE STRICTLY CONVEX BODIES WITH POSITIVE TOPOLOGICAL ENTROPY

José Pedro Gaivão
University of Lisbon, Portugal
Joint work with **M. Bessa, G. Del Magno, J.L. Dias, and M.J. Torres**.

In this talk we discuss the topological entropy of billiards inside strictly smooth convex bodies. We show that in a C^2-open and dense set of strictly convex bodies, the associated multidimensional billiard maps have positive topological entropy.

WILD PSEUDOHYPERBOLIC ATTRACTOR IN A FOUR-DIMENSIONAL LORENZ MODEL

Alexey O. Kazakov
National Research University Higher School of Economics, Russia
Joint work with **Sergey Gonchenko and Dmitry Turaev**.

We present an example of a new strange attractor. We show that it belongs to a class of wild pseudohyperbolic spiral attractors. A theory of pseudohyperbolic spiral attractors was proposed in [1], however examples of concrete systems of differential equations with such attractors were not known.

We consider the following system of differential equation

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = x(r - z) - y, \\ \dot{z} = xy - bz + \mu w, \\ \dot{w} = -bw - \mu z, \end{cases} \tag{1}$$

where σ, r, b and μ are parameters. This system can be viewed as a four-dimensional extension of the classical Lorenz model. We perform a series of numerical experiments with the strange

attractor which exists in the system at $\mu = 7, \sigma = 10, b = 8/3, r = 25$. We demonstrate that this attractor is indeed pseudohyperbolic and wild.

The pseudohyperbolicity [1, 2] is a key word here. It means that certain conditions hold which guarantee that every orbit in the attractor is unstable (i.e. it has a positive maximal Lyapunov exponent). Moreover, this instability property persists for all small perturbations of the system.

The wildness of the observed attractor means that it contains a "wild hyperbolic set" [3 ,4] - a uniformly hyperbolic invariant set which has a pair of orbits such that the unstable manifold of one orbit has a nontransversal intersection with the stable manifold of the other orbit in the pair and this property is preserved for all C^2-small perturbations.

Acknowledgments

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SPECTRAL DETERMINATION OF OPEN DISPERSING BILLIARDS

Martin Leguil
University of Toronto, Canada
Joint work with **Péter Bálint, Jacopo De Simoi, and Vadim Kaloshin**.

In an ongoing project with P. Bálint, J. De Simoi and V. Kaloshin, we have been studying the inverse problem for a class of open dispersing billiards obtained by removing from the plane a finite number of smooth strictly convex scatterers satisfying a non-eclipse condition. The restriction of the dynamics to the set of non-escaping orbits is conjugated to a subshift of finite type that provides a natural labeling of all periodic orbits. One direction we have investigated in [1] is whether it is possible to recover from the Marked Length Spectrum (i.e., the set of lengths of all periodic orbits together with their labeling) the local geometry near periodic points. In particular, we show in [1] that the Marked Length Spectrum determines the curvatures of the scatterers at the base points of 2-periodic orbits, and the Lyapunov exponents of each periodic orbit. In a second work [2], we show that it is generically possible, in the analytic category and for billiard tables with two (partial) axial symmetries, to determine completely the geometry of those billiards from the purely dynamical data encoded in their Marked Length Spectrum.

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PERSISTENT HETERODIMENSIONAL CYCLES IN PERIODIC PERTURBATIONS OF LORENZ-LIKE ATTRACTORS

Dongchen Li
Imperial College London, UK
Joint work with **Dmitry Turaev**.

We prove that heterodimensional cycles can be created by unfolding a pair of homoclinic tangencies in a certain class of C^∞ diffeomorphisms. This implies the existence of a C^2-open domain in the space of dynamical systems with a certain type of symmetry where systems with heterodimensional cycles are dense in C^∞. In particular, we describe a class of three-dimensional

flows with a Lorenz-like attractor such that an arbitrarily small perturbation of any such flow can belong to this domain - in this case the corresponding heterodimensional cycles belong to a chain-transitive attractor of the perturbed flow.

ON HAUSDORFF DIMENSION OF THIN NONLINEAR SOLENOIDS

Reza Mohammadpour
Polish Academy of Sciences, Poland
Joint work with **Michał Rams and Feliks Przytycki**.

Let $M = S^1 \times \mathbb{D}$ be the solid torus, where $\mathbb{D} = \{v \in \mathbb{R}^2 || v| < 1\}$ carries the product distance $d = d_1 \times d_2$, and suppose $f : M \to M$ such that

$$(x, y, z) \mapsto (\eta(x, y, z) \bmod 2\pi, \lambda(x, y, z) + u(x), \nu(x, y, z) + v(x)) \tag{1}$$

is a smooth embedding map.

Bothe [1] was the first who obtained results on the dimension of the attractor of a thin linear solenoid where contraction rates are strong enough. Barriera, Pesin and Schemeling [2] established a dimension product structure of invariant measures in the course of proving the Eckmann Ruelle conjecture.

Conjecture. The fractal dimension of a hyperbolic set is (at least generically or under mild hypotheses) the sum of those of its stable and unstable slices, where fractal can mean either Hausdorff or upper box dimension.

In spite of the difficulties due to possible low regularity of the holonomies, indeed, Schmeling [4] found that solenoids often lack regular holonomies but the set of non-liptchitz points seemed to be rather small in the measure scene. Hasselblat and Schmeling [3] proved the conjecture for a class of thin linear solenoids. We prove the conjecture for a class of thin nonlinear solenoids of map (1).

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THE LOCAL LIMIT THEOREM FOR HYPERBOLIC DYNAMICAL SYSTEMS AND APPLICATIONS

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We present a convenient joint generalization of mixing and the local version of the central limit theorem (MLLT) for probability preserving dynamical systems. We verify that the MLLT holds for several examples of hyperbolic systems by reviewing old results for maps and presenting new results for flows. Examples include reward renewal processes, Axiom A flows, as well as the systems admitting Young's tower, such as Sinai's billiard with finite horizon, suspensions over Pomeau-Manneville maps and geometric Lorenz attractors. Then we discuss two applications in infinite ergodic theory. First, we prove the mixing of global observables by some infinite measure preserving hyperbolic systems that are well approximated by periodic systems (examples include billiards with small potential field and various ping pong models). Here, global observables are functions that are not integrable with respect to the infinite invariant measure, but have convergent average values over large boxes. Second, we discuss the Birkhoff theorem for such global observables in the simplest case: iid random walks. The talk is based on joint work with Dmitry Dolgopyat and in parts with Marco Lenci.

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BIRTH OF DISCRETE LORENZ ATTRACTORS IN GLOBAL BIFURCATIONS

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Discrete Lorenz attractors are chaotic attractors, which are the discrete-time analogues of the well-known continuous-time Lorenz attractors. They are genuine strange attractors, i.e. they do not contain simpler regular attractors such as stable equilibria, periodic orbits etc. In addition, this property is preserved under small perturbations. Thus, Lorenz attractors, discrete and continuous, represent the so-called robust chaos.

In the talk a list of global (homoclinic and heteroclinic) bifurcations [1, 2, 3, 4] is presented, in which it was possible to prove the appearance of discrete Lorenz attractors. The proof is based on the study of first return (Poincare) maps, which are defined in a small neighbourhood of the homoclinic or heteroclinic cycle. The first return map can be transformed to the form asymptotically close to the three-dimensional Hénon map via smooth transformations of coordinates and parameters. According to [1, 5, 6, 7], Henon-like maps possess the discrete Lorenz attractor in an open subset of the parameter space.

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THE ROVELLA ATTRACTOR IS ASYMPTOTICALLY SECTIONAL-HYPERBOLIC

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Joint work with **Kendry Vivas**.

The Rovella attractor is a compact invariant set for a vector field X_0 constructed in a similar way as the geometric Lorenz attractor, but replacing the central expansive condition at the singularity by a central contracting condition plus two additional geometric hypothesis: the unstable manifold of the singularity is contained in the stable manifold of hyperbolic periodic orbits and the one dimensional reduction for the first return Poincaré map has negative Schwarzian derivative. Rovella showed that although this attractor is non robust, it is almost 2-persistent in the C^3 topology. In this paper we will prove that for a generic two-parameter family of vector fields that contains X_0, asymptotically sectional-hyperbolicity is an almost 2-persistent property. In particular, we will prove that the Rovella attractor is asymptotically sectional-hyperbolic.

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INSTABILITY AND DIFFUSION IN HAMILTONIAN SYSTEMS VIA THE APPROXIMATION BY CONJUGATION METHOD

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Joint work with **Bassam Fayad**.

We present examples of nearly integrable Hamiltonian systems with several strong diffusion properties. In particular, we construct a real-analytic near integrable Hamiltonian system whose flow is topologically weakly mixing on the energy surface.

Our constructions are obtained by a version of the successive conjugation scheme *à la* Anosov-Katok. The talk is based on a joint work with Bassam Fayad.

LORENZ ATTRACTORS IN FLOWS AND MAPS

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We review a theory of pseudohyperbolic attractors, which serve as a generalization of the classical Lorenz attractor to the case of a higher codimension of the strong-stable foliation. These attractors are genuinely chaotic (every orbit in such attractor has positive maximal Lyapunov exponent) and are robust with respect to small perturbations. The class includes periodically perturbed hyperbolic and Lorenz attractors and attractors in lattice dynamical systems. We show that pseudohyperbolic attractors emerge naturally at local and global bifurcations of codimension 3, hence they are present in a vast set of diverse applications. We also show that robust presence of homoclinic tangencies and heterodimensional cycles is a characteristic feature of pseudohyperbolic attractors.

CONJUGACY CLASSES OF REAL ANALYTIC MAPS: ON A QUESTION OF AVILA-LYUBICH-DE MELO

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Joint work with **Trevor Clark**.

Avila-Lyubich-de Melo proved that the topological conjugacy classes of unimodal real-analytic maps are complex analytic manifolds, which laminate a neighbourhood of any such mapping without a neutral cycle. Their proof that the manifolds are complex analytic depends on the fact that they have codimension-one in the space of unimodal mappings.

In joint work with Trevor Clark, we show how to construct a “pruned polynomial-like mapping" associated to a real mapping. This gives a new complex extension of a real-analytic mapping.

The additional structure provided by this extension, makes it possible to generalize this result

of Avila-Lyubich-de Melo to interval mappings with several critical points. Thus we show that the conjugacy classes are complex analytic manifolds whose codimension is determined by the number of critical points.

Building on these ideas, we will show that in the space of unimodal mappings with negative Schwarzian derivative, the conjugacy classes laminate a neighbourhood of every mapping.

CLASSIFICATION OF TOTALLY CARTAN ACTIONS

Kurt Vinhage
University of Chicago, USA
Joint work with **Ralf Spatzier**.

We will discuss recent progress on the Katok-Spatzier conjecture, which aims to classify Anosov $\mathbb{R}^k$ and $\mathbb{Z}^k$ actions under the assumption that there are no nontrivial smooth rank one factors. Classification is the strongest conclusion in the smooth rigidity program, which assumes nothing about the structure of the underlying manifold or dynamics other than the Anosov hyperbolicity assumptions. We develop new techniques to build homogeneous structures from dynamical ones. The remarkable features of the techniques are their low regularity requirements and their use of metric geometry over differential geometry to build group actions. We apply these techniques to the totally Cartan setting, where bundles associated to the hyperbolic strcuture are one-dimensional. Joint with Ralf Spatzier.

KAKUTANI EQUIVALENCE OF UNIPOTENT FLOWS

Daren Wei
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Joint work with **Adam Kanigowski and Kurt Vinhage**.

We study Kakutani equivalence in the class of unipotent flows acting on finite volume quotients of semisimple Lie groups. For every such flow we compute the Kakutani invariant of M. Ratner, the value of which being explicitly given by the Jordan block structure of the unipotent element generating the flow. This, in particular, answers a question of M. Ratner. Moreover, it follows that the only standard unipotent flows are given by $\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \times \mathrm{id}$ acting on $(\mathrm{SL}(2,\mathbb{R}) \times G')/\Gamma'$, where Γ' is an irreducible lattice in $\mathrm{SL}(2,\mathbb{R}) \times G'$ (with the possibility that $G' = \{e\}$).

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ON THE RIGIDITY OF PARTIALLY HYPERBOLIC $\mathbb{Z}^k$ ACTION

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Roughly speaking, a partially hyperbolic diffeomorphism on a manifold M is a certain type of mapping, from M to itself, with local directions of "expansion", "neutral" and "contraction". The study of the partially hyperbolic system, i.e. the $\mathbb{Z}$-action generated by the iterates of a partially hyperbolic diffeomorphism, is one of the central topic in dynamical systems in the last four decades.

On the other hand, in general it is expected that under suitable assumptions, a $\mathbb{Z}^k$ action by diffeomorphisms on manifold, $k > 1$ shares some strong rigidity properties (stronger than that of $\mathbb{Z}$-action). A $\mathbb{Z}^k$-action on a manifold is called partially hyperbolic if the action contains at least one partially hyperbolic diffeomorphism. In this talk we will show some recent rigidity results on the study of partially hyperbolic $\mathbb{Z}^k$-action on manifolds, this is a joint work with D. Damjanović and A. Wilkinson.