DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D11

AGH University of Science and Technology

Kraków, Poland

16–20 September 2019

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D1

THE FRACTIONAL SUSCEPTIBILITY FUNCTION FOR THE QUADRATIC FAMILY

Viviane Baladi
CNRS & Sorbonne Université, France
Joint work with **Daniel Smania**.

For t in a set Ω of positive measure, maps in the quadratic family $f_t(x) = t - x^2$ admit an SRB measure μ_t. On the one hand, the dependence of μ_t on t has been shown [1] to be no better than 1/2 Hölder, on a subset of Ω, for t_0 a suitable Misiurewicz-Thurston parameter. On the other hand, the susceptibility function $\Psi_t(z)$, whose value at $z = 1$ is a candidate for the derivative of μ_t with respect to t, has been shown [2] to admit a holomorphic extension at $z = 1$ for $t = t_0$. Our goal is to resolve this paradox. For this, we introduce and study a fractional susceptibility function.

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UNIQUE ERGODICITY FOR FOLIATIONS ON COMPACT KAEHLER SURFACES

Tien-Cuong Dinh
National University of Singapore, Singapore
Joint work with **Viet-Anh Nguyen and Nessim Sibony**.

Let F be a holomorphic foliation by Riemann surfaces on a compact Kaehler surface. Assume it is generic in the sense that all the singularities are hyperbolic and that the foliation admits no directed positive closed $(1,1)$-current, or equivalently, no invariant measure. Then there exists a unique (up to a multiplicative constant) positive ddc-closed $(1,1)$-current directed by F, or equivalently, a unique harmonic measure. This is a very strong ergodic property showing that all leaves of F have the same asymptotic behavior. Our proof uses an extension of the theory of densities to a new class of currents. A complete description of the cone of directed positive ddc-closed $(1,1)$-currents (i.e. harmonic measures) is also given when F admits directed positive closed currents (i.e. invariant measures).

MEASURE RIGIDITY FOR HIGHER RANK DIAGONALIZABLE ACTIONS

Manfred Einsiedler
ETH Zürich, Switzerland
Joint work with **Elon Lindenstrauss**.

We review old and recent measure rigidity results for higher rank diagonalizable actions on homogeneous spaces and contrast these results with the rank one and unipotent case. After this we consider higher rank actions on irreducible arithmetic quotients of $\mathrm{SL}_2(\mathbb{R})^k$ for $k \geq 2$. If the quotient is compact, positive entropy of an ergodic invariant measure μ implies algebraicity of μ with semisimple stabiliser. For non-compact quotients there are more possibilities. The main novelty here is that the acting group does not have to be maximal or in a special position. The main new idea is to use a quantitative recurrence phenomenon to transport positivity of entropy for one acting element to another.

EQUIDISTRIBUTION FOR COMMUTING MAPS

Michael Hochman
Hebrew University of Jerusalem, Israel

In two classical papers circa 1960, J. Cassels and W. Schmidt proved that a.e. numbers in the ternary Cantor set (with respect to Cantor-Lebesgue measure) eqidistributes for Lebesgue measure under the map $Tx = bx$ mod 1, whenever b is an integer that is not a power of 3. This phenomenon has since been established in much greater generality on the interval, e.g. Host's theorem, according to which one can replace Cantor-Lebesgue measure by any $\times 3$-ergodic measure of positive entropy, provided $\gcd(3, b) = 1$. In this talk I will describe a new and heuristically simple proof of such results, and then discuss how it can be extended to give new results in multi-dimensional settings.

ON DYNAMICAL SPECTRAL RIGIDITY OF PLANAR DOMAINS

Vadim Kaloshin
University of Maryland, College Park, USA

Consider a convex domain on the plane and the associated billiard inside. The length spectrum is the closure of the union of perimeters of all period orbits. The length spectrum is closely related to the Laplace spectrum, through so-called the wave trace. The well-known question popularized by M. Kac: "Can you hear the shape of a drum?" asks if the Laplace spectrum determines a domain up to isometry. We call a domain dynamically spectrally rigid (DSR) if any smooth deformation preserving the length spectrum is an isometry. During the talk I will discuss recent results on DSR of convex planar domains.

ON THE DIVERGENCE OF BIRKHOFF NORMAL FORMS

Raphaël Krikorian
CNRS & Université de Cergy-Pontoise, France

A real analytic hamiltonian or a real analytic exact symplectic diffeomorphism admitting a non resonant elliptic fixed point is always formally conjugated to a formal integrable system, its Birkhoff Normal Form (BNF). Siegel proved in 1954 that the formal conjugation reducing a hamiltonian to its BNF is in general divergent and Hakan Eliasson has asked whether the BNF itself could be divergent. Perez-Marco proved in 2001 that for any fixed non resonant frequency vector the following dichotomy holds: either any real analytic hamiltonian system admitting this frequency vector at the origin has a convergent BNF or for a prevalent set of hamiltonians admitting this frequency vector the BNF generically diverges. It is possible to exhibit examples of hamiltonian systems with diverging BNF (X. Gong 2012 or the recent examples of B. Fayad in 4 degrees of freedom). The aim of this talk is to give a complete answer to the question of the divergence of the BNF (in the setting of exact symplectic diffeomorphisms): for any non resonant frequency vector, the BNF of a real analytic exact symplectic diffeomorphism admitting this frequency vector at the origin, is in general divergent. This theorem is the consequence of the remarkable fact that the convergence of the formal object that is the BNF has dynamical consequences, in particular an abnormal abundance of invariant tori.

KINETIC THEORY FOR THE LOW-DENSITY LORENTZ GAS

Jens Marklof
University of Bristol, UK
Joint work with **Andreas Strombergsson**.

The Lorentz gas is one of the simplest and most widely-studied models for particle transport in matter. It describes a cloud of non-interacting gas particles in an infinitely extended array of identical spherical scatterers, whose radii are small compared to their mean separation. The model was introduced by Lorentz in 1905 who, following the pioneering ideas of Maxwell

and Boltzmann, postulated that its macroscopic transport properties should be governed by a linear Boltzmann equation. A rigorous derivation of the linear Boltzmann equation from the underlying particle dynamics was given, for random scatterer configurations, in three seminal papers by Gallavotti, Spohn and Boldrighini-Bunimovich-Sinai. The objective of this lecture is to develop an approach for a large class of deterministic scatterer configurations, including various types of quasicrystals. We prove the convergence of the particle dynamics to transport processes that are in general (depending on the scatterer configuration) not described by the linear Boltzmann equation. This was previously understood only in the case of the periodic Lorentz gas through work of Caglioti-Golse and Marklof-Strombergsson. Our results extend beyond the classical Lorentz gas with hard sphere scatterers, and in particular hold for general classes of spherically symmetric finite-range potentials. We employ a rescaling technique that randomises the point configuration given by the scatterers' centers. The limiting transport process is then expressed in terms of a point process that arises as the limit of the randomised point configuration under a certain volume-preserving one-parameter linear group action.

INFLECTION POINTS FOR LYAPUNOV SPECTRA

Mark Pollicott
University of Warwick, UK
Joint work with **Oliver Jenkinson and Polina Vytnova**.

The Lyapunov spectra for a dynamical system describes the size (Hausdorff dimension) of the set of points which have a given Lyapunov exponent. H. Weiss conjectured that the associated graph is convex, but Iommi and Kiwi constructed a simple counter example. We explore this problem further, constructing examples with any given number of points of inflection.

MANDELBROT SET SEEN BY HARMONIC MEASURE: THE SIMILARITY MAP

Grzegorz Świątek
Warsaw University of Technology, Poland
Joint work with **Jacek Graczyk**.

We study conformal quantities at generic parameters with respect to the harmonic measure on the boundary of the connectedness loci $\mathcal{M}_d$ for unicritical polynomials $f_c(z) = z^d + c$. It is known that these parameters are structurally unstable and have stochastic dynamics. In [3] it was shown that for c from a set of full harmonic measure in $\partial\mathcal{M}_d$ there exists a quasi-conformal similarity map Υ_c between phase and parameter spaces which is conformal at c. In a recent work [2] we prove $C^{1+\frac{\alpha}{d}-\epsilon}$-conformality, $\alpha = \mathrm{HD}(\mathcal{J}_c)$, of $\Upsilon_c(z) : \mathbb{C} \mapsto \mathbb{C}$ at typical $c \in \partial\mathcal{M}_d$ and establish that globally quasiconformal similarity maps $\Upsilon_c(z)$, $c \in \partial\mathcal{M}_d$, are C^1-conformal along external rays landing at c in $\mathbb{C} \setminus \mathcal{J}_c$ mapping onto the corresponding rays of $\mathcal{M}_d$. This conformal equivalence leads to a proof that the z-derivative of the similarity map $\Upsilon_c(z)$ at typical $c \in \partial\mathcal{M}_d$ is equal to $1/\mathcal{T}'(c)$, where

$$\mathcal{T}(c) = \sum_{n=0}^{\infty} \left(D_z \left[f_c^n(z)\right]_{z=c}\right)^{-1}$$

is the transversality function previously studied by Benedicks-Carleson and Levin, see [1, 4]. There are additonal geometric consequences of these results. A typical external radius of the connectedness locus is contained in an asymptotically very nearly linear twisted angle, but nevertheless passes through infinitely many increasingly narrow straits.

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TALKS OF SESSION D11

ALL MINIMAL CANTOR SYSTEMS ARE SLOW

Jan Boroński
AGH University of Science and Technology, Poland
Joint work with **Jiří Kupka and Piotr Oprocha**.

A Cantor set is a 0-dimensional compact metric space without isolated points, and Cantor system is a dynamical system on the Cantor set. A minimal system is the one that has all orbits dense. We are interested in the following question: *Can every minimal Cantor system be embedded into* $\mathbb{R}$ *with vanishing derivative everywhere?* A particular instance of that question was raised by Samuel Petite at the *Workshop on Aperiodic Patterns in Crystals, Numbers and Symbols* that took place in Lorentz Center in June of 2017, who asked if expansive minimal Cantor systems have this property. It was conjectured that the expansive systems lack such a property, because some kind of expanding must take place in these systems. In contrast, I shall discuss a postive answer to the above question on all Cantor minimal systems, obtained in [2]. There are more reasons for which this result seems surprising. By the Margulis-Ruelle inequality the topological entropy of a piecewise Lipschitz differentiable map f, with an invariant measure μ, is bounded from above by the integral over the support of μ of the Lyapunov characteristic of f. In the case of derivative zero, all Lyapunov exponents, and as a result Lyapunov characteristic of f are all equal to 0. Therefore it is natural to expect that vanishing derivative on an invariant set will imply zero entropy on that set. Such an intuition was supported by the zero entropy examples in [3] and [1]. However our result shows that no such connection exists. Note that if f is C^1 on $\mathbb{R}$ and $f(P) \subset P$ for some perfect compact subset $P \subset \mathbb{R}$, then there is $x \in P$ with $|f'(x)| \geq 1$ (see [4] for a nice survey on this and related topics). For systems with positive entropy it is also a consequence of Margulis-Ruelle inequality mentioned above, so in this case the map is not even Lipschitz continuous.

This gives rise to the following question: *Can the differentiable extensions of minimal Cantor systems to $\mathbb{R}$, guaranteed by our result, be additionally required to be α-Hölder continuous for some $0 < \alpha < 1$?*

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ON UNIMODAL INVERSE LIMIT SPACES

Henk Bruin
Universität Wien, Austria

Inverse limit spaces of unimodal maps are used to model attractors of maps (on the plane). As such, the classification of unimodal inverse limit spaces has an impact on out understanding of e.g. Henon-like attractors. In this talk I want to give an update on how topological properties of unimodal inverse limit spaces relate to dynamical properties of the underlying map.

PRIME ENDS DYNAMICS IN PARAMETRISED FAMILIES OF ROTATIONAL ATTRACTORS

Jernej Činč
AGH University of Science and Technology, Poland & University of Ostrava, Czech Republic
Joint work with **Jan P. Boroński and Xiao-Chuan Liu** .

The prime ends rotation number induced by surface homeomorphisms restricted to open domains is one of the important tools in the study of boundary dynamics. Parametrised families of dynamical systems can provide a clearer view of both the surface dynamics and the boundary dynamics in many situations. Our study [1] serves as a contribution in this direction, by providing new examples in various contexts, by investigating the prime ends rotation numbers arising from parametrized BBM embeddings of inverse limits of topological graphs [2].

First, motivated by a topological version of the Poincaré-Bendixson Theorem obtained recently by Koropecki and Passeggi [4], we show the existence of homeomorphisms of S^2 with Lakes of Wada rotational attractors, with an arbitrarily large number of complementary domains, and with or without fixed points, that are arbitrarily close to the identity. This answers a question of Le Roux.

Second, with the help of a reduced Arnold's family we construct a parametrised family of Birkhoff-like cofrontier attractors, where except for countably many choices of the parameters, two distinct irrational prime ends rotation numbers are induced from the two complementary domains. This contrasts with the negative resolution of Walker's Conjecture from [5] by Koropecki, Le Calvez and Nassiri [3], and implies that our examples induce Denjoy homeomorphisms on the circles of prime ends.

Third, answering a question of Boyland, we show that there exists a non-transitive Birkhoff-like attracting cofrontier which is obtained from a BBM embedding of inverse limit of circles, such that the interior prime ends rotation number belongs to the interior of the rotation interval of the cofrontier dynamics. There exists another BBM embedding of the same attractor so that the two induced prime ends rotation numbers are exactly the two endpoints of the rotation interval.

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BEYOND TOPOLOGICAL HYPERBOLICITY

Welington Cordeiro

Polish Academy of Sciences, Poland

We discuss the dynamics beyond topological hyperbolicity considering homeomorphisms satisfying the shadowing property and generalizations of expansivity. First of all, we will talk about some of these generalizations of expansivity and show some interesting examples. In particular, we will define *entropy expansivity*, N*-expansivity* and *measure expansivity* and we will give a overview about the recent results for these systems.

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SOME APPLICATIONS OF LOCAL ENTROPY THEORY

Udayan B. Darji
University of Louisville, USA

In this talk we discuss some applications of local entropy theory, in particular results of Kerr-Li, to problems in topological dynamics and induced dynamics on the space of probability measures.

In the setting of topological dynamics, we discuss how local entropy theory can be used to show that in certain settings, the complexity of a dynamical system implies indecomposability in the inverse limit space of the dynamical system [2, 3], settling some old problems stated in [1].

A topological dynamical system (X, f) induces natural dynamics on $P(X)$, the space of probability measure on X defined by $\tilde{f} : P(X) \to P(X)$ by $\tilde{f}(\mu) = \mu f^{-1}$. A nontrivial and a remarkable result is that f has topological entropy zero if and only if $\tilde{f}$ has measure zero [4]. Using techniques of [4], recently it was shown [5] that one can sharpen this result to null systems. We discuss how local entropy theory can be used to prove theorems of these types with ease.

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PRODUCT-MINIMAL SPACES

Matúš Dirbák
Matej Bel University, Slovakia
Joint work with **Ľubomír Snoha and Vladimír Špitalský**.

We call a compact metric space Y product-minimal (respectively, homeo-product-minimal) if for every minimal system (X, T) there is a continuous map (respectively, a homeomorphism) $S\colon Y \to Y$ such that the product system $(X \times Y, T \times S)$ is minimal. Every homeo-product-minimal space is product-minimal and every product-minimal space is minimal, while the converse implications do not hold. In the talk we shall present examples of (homeo-)product-minimal spaces and list some operations, under which the class of all (homeo-)product-minimal spaces is closed.

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TOPOLOGICAL AND COMBINATORIAL PROPERTIES OF FINITE RANK MINIMAL SUBSHIFTS

Sebastián Donoso
University of Chile, Chile
Joint work with **Fabien Durand, Alejandro Maass, and Samuel Petite**.

I will discuss topological and combinatorial properties of finite rank minimal systems, establishing a clear connection with the S-adic subshifts, under recognizability assumptions. I will also mention results concerning the asymptotic components of a finite rank subshift and show that there is a rank two minimal subshift with superlinear complexity. I will mention results concerning the automorphism group of a finite rank subshift and state some open questions.

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FACTORING GROUP SHIFTS ONTO THE FULL SHIFT

Bartosz Frej
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Joint work with **Dawid Huczek**.

It is known that any subshift of finite type with the action of $\mathbb{Z}$ and entropy greater or equal than $\log n$ factors onto the full shift over n symbols (see [7] and [1] for the cases of equal and unequal entropy, respectively). Extending these results for actions of other groups has been difficult, and it is known that a factor map onto a full shift of equal entropy may not exist in this case (see [2]). Johnson and Madden showed in [6] that any SFT with the action of $\mathbb{Z}^d$, which has entropy greater than $\log n$ and satisfies an additional mixing condition (known as corner gluing), has an extension which is finite-to-one (hence of equal entropy) and maps onto the full shift over n symbols. This result was later improved by Desai in [4] and finally by Boyle, Pavlov and Schraudner in [3].

I will prove that in the case of actions of a countable amenable group, any strongly irreducible symbolic dynamical system with entropy greater than $\log n$ has an equal-entropy symbolic extension which factors onto the full shift over n symbols. The construction uses tilings of amenable groups as presented in [5].

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TAME IMPLIES REGULAR

Gabriel Fuhrmann
Imperial College London, UK
Joint work with **Eli Glasner, Tobias Jäger, and Christian Oertel**.

The last decade saw an increased interest in tame systems revealing their connections to different areas of mathematics like Banach spaces, substitutions and tilings, quasicrystals, cut and project schemes and even model theory and logic. A major breakthrough in the general understanding of tameness was achieved by Glasner's recent structural result for tame minimal systems [1]. One of its consequences is that a tame minimal dynamical system which has an invariant measure is almost automorphic, uniquely ergodic and measure-theoretically isomorphic to its maximal equicontinuous factor.

In this talk, we prove that tame minimal dynamical systems (X, G) with an invariant measure are actually regularly almost automorphic, that is, they allow for a factor map π from (X, G) to an equicontinuous system $(\mathbb{T}, G)$ such that almost every point in $\mathbb{T}$ (with respect to the unique invariant measure on $\mathbb{T}$) has a unique preimage under π, see [2].

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TOPOLOGICAL MODELS OF KRONECKER SYSTEMS

Felipe García-Ramos
Autonomous University of San Luis Potosí, Mexico
Joint work with **Tobias Jäger, Xiangdong Ye, and Dominik Kwietniak**.

I will talk about the range of behaviours of topological models of Kronecker systems (a.k.a. discrete spectrum systems) and loosely Kronecker systems (a.k.a. zero entropy loosely Bernoulli).

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A PROBABILISTIC TAKENS THEOREM

Yonatan Gutman
Polish Academy of Sciences, Poland
Joint work with **Krzysztof Barański and Adam Śpiewak**.

Let $X \subset \mathbb{R}^N$ be a Borel set, μ a Borel probability measure on X and $T : X \to X$ a Lipschitz and injective map. Fix $k \in \mathbb{N}$ greater than the (Hausdorff) dimension of X and assume that the set of p-periodic points has dimension smaller than p for $p = 1, \ldots, k-1$. We prove that for a typical polynomial perturbation $\tilde{h}$ of a given Lipschitz map $h : X \to \mathbb{R}$, the k-delay coordinate map $x \mapsto (\tilde{h}(x), \tilde{h}(Tx), \ldots, \tilde{h}(T^{k-1}x))$ is injective on a set of full measure μ. This is a probabilistic version of the Takens delay embedding theorem as proven by Sauer, Yorke and Casdagli. We also provide a non-dynamical probabilistic embedding theorem of similar type, which strengthens a previous result by Alberti, Bölcskei, De Lellis, Koliander and Riegler. In both cases, the key improvements compared to the non-probabilistic counterparts are the reduction of the number of required measurements from $2 \dim X$ to $\dim X$ and using Hausdorff dimension instead of the box-counting one. We present examples showing how the use of the Hausdorff dimension improves the previously obtained results and settle conjectures in the physics literature.

DISTRIBUTIONAL CHAOS IN RANDOM DYNAMICAL SYSTEMS

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Consider two continuous interval maps $f, g : [0,1] \to [0,1]$ and the random dynamical system given by

$$x_{n+1} = \begin{cases} f(x_n) & \text{with probability } p, \\ g(x_n) & \text{with probability } 1-p, \end{cases}$$

where $p \in (0,1)$. Distributional chaos for such systems was defined in [1]. We will discuss some of its properties (for example stability, types of distributional chaos, etc.).

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A MEASURE PRESERVING PL MODIFICATION OF THE JONES-YORKE BOUNDED ORBIT FLOW ON $\mathbb{R}^3$

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Joint work with **Jeffrey Ford**.

We present a modification of the example by G. Stephen Jones and James A. Yorke of a smooth, bounded orbit, dynamical system on $\mathbb{R}^3$. Our example is piecewise linear and measured. We use methods of piecewise linear dynamical system developed by Greg Kuperberg, in particular his notion of a slanted suspension.

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ON THE ABUNDANCE OF (NON)TAME GROUP ACTIONS

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Tame dynamical systems were introduced by Köhler [3] in 1995. Tameness is a topological notion roughly corresponding to compactness appearing in the compactness vs weak mixing dichotomy that underlines the structure theory for measure preserving actions.

In recent years several authors developed the theory of tame systems revealing connections to other areas of mathematics like Banach spaces, circularly ordered systems, substitutions and tilings, quasicrystals, cut and project schemes and even model theory and logic.

During my talk, I will discuss results of [1], where we study tameness and nullness of regular almost automorphic G-actions utilising a generalised notion of semi-cocycle extensions. In particular, we show that every ergodic equicontinuous G-action on a compact metric space admits a regular almost automorphic extension which is non-tame as well as tame but non-null extension. In some sense, this complements a recent result of Glasner [2]. We prove that such examples appear in well-studied families of group actions including Delone dynamical systems and symbolic systems (including Toeplitz flows over arbitrary G-odometers).

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RECENT DEVELOPMENTS ON MEAN EQUICONTINUITY AND MEAN SENSITIVITY

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In this talk we will discuss recent developments on mean equicontinuity and mean sensitivity both on topological and measure-theoretic settings, based on the following three papers and related works.

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RENORMALIZATION TOWERS AND THEIR FORCING

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Joint work with **Alexander Blokh**.

Over half a century ago, Sharkovsky proved his theorem about periodsof periodic orbits of continuous interval maps. Existence of someperiods force existence of some other periods, and the orderingobtained in such a way is linear. Later, people noticed that insteadof looking at periods, one can take into account permutations.Unfortunately, this gives only a partial order, which is verycomplicated and impossible to describe in simple terms. We propose themiddle ground: to look at the block structures of permutations (theycan be also understood in terms of renormalizations). This is a finerclassification of periodic orbits than just by periods, but stillresults in a linear ordering.

ON THE ENTROPY CONJECTURE OF MARCY BARGE

Piotr Oprocha
AGH University of Science and Technology, Poland
Joint work with **Jan Boroński and Jernej Činč**.

I shall discuss a positive solution to the following problem, obtained in a joint work with J. Boroński and J. Činč.

Question (M. Barge, 1989 [8]) Does there exist, for every $r \in [0, \infty]$, a pseudo-arc homeomorphism whose topological entropy is r?

Until now all known pseudo-arc homeomorphisms have had entropy 0 or ∞. Recall that the pseudo-arc is a compact and connected space (continuum) first constructed by Knaster in 1922 [6]. It can be seen as a pathological fractal. According to the most recent characterization [5] it is topologically the only, other than the arc, continuum in the plane homeomorphic to each of its proper subcontinua. The pseudo-arc is homogeneous [2] and played a crucial role in the classification of homogeneous planar compacta [4]. Lewis showed that for any n the pseudo-arc admits a period n homeomorphism that extends to a rotation of the plane, and that any P-adic Cantor group action acts effectively on the pseudo-arc [7] (see also [10]). We adapt Lewis' inverse limit constructions, by combining them with a Denjoy-Rees scheme [1] (see also [9], [3]). The positive entropy homeomorphisms that we obtain are periodic point free, except for a unique fixed point.

I am going to present various results related to the problem, to conclude with a discussion of its solution.

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NON-UNIFORM SPECIFICATION PROPERTIES ON SUBSHIFTS

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A celebrated result of Bowen implies uniqueness of equilibrium state for certain potentials on expansive systems with the specification property. In the setting of symbolic dynamics, this property is equivalent to the existence of a constant N such that any two n-letter words v, w in the language can be combined into a new word in the language given a gap between them of length at least N. Several weakenings of specification have seen recent activity, among them non-uniform specification, in which one allows the gap to have size controlled by an increasing function $f(n)$. I will summarize some known results about non-uniform specification, including a provable threshold on $f(n)$ below which one can prove generalizations of Bowen's result on uniqueness of equilibrium state.

References

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REMARKS ON DEFINITIONS OF PERIODIC POINTS FOR NONAUTONOMOUS DYNAMICAL SYSTEM

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Let $(X, f_{1,\infty})$ be a nonautonomous dynamical system. In this talk we summarize known definitions of periodic points for general nonautonomous dynamical systems and propose a new definition of asymptotic periodicity. This definition is not only very natural but also resistant to changes of a beginning of the sequence generating the nonautonomous system. We show the relations among these definitions and discuss their properties. We prove that for uniformly convergent nonautonomous systems topological transitivity together with dense set of asymptotically periodic points imply sensitivity. We also show that even for uniformly convergent systems the nonautonomous analog of Sharkovsky's Theorem is not valid for most definitions of periodic points.

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ON THE PROBLEM OF THE TOPOLOGICAL CLASSIFICATION OF MINIMAL SETS

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One of the open problems in topological dynamics is the problem of the topological classification of minimal spaces/sets.

A dynamical system (X, f) given by a topological space X and a continuous map $f : X \to X$ is called *minimal* if all forward orbits are dense. A nonempty closed set $M \subseteq X$ with $f(M) \subseteq M$ is a *minimal set* for the system (X, f) or for the map f, if $(M, f|_M)$ is a minimal system. Thus, a system (X, f) is minimal if and only if X is a minimal set. In every compact system there are minimal sets. A space X is said to be *minimal* if it admits a minimal (in general noninvertible) map.

The classification problem has two parts:

(1) Which spaces X are minimal and which are not?

(2) Given a space X, consider all possible continuous selfmaps of this space and their minimal sets. Describe these sets topologically (find their full topological characterization).

(An illustrating example to (2): The minimal sets in $I = [0,1]$ are nonempty finite sets and Cantor sets, meaning that if $f\colon I \to I$ is continuous and M is a minimal set of f, then M is either nonempty finite or Cantor and, conversely, if $M \subseteq I$ is nonempty finite or Cantor, then there is a continuous map $f\colon I \to I$ such that M is a minimal set of f.)

In the talk we give a survey of some results on the classification problem.

MINIMAL INTERVAL EXCHANGE TRANSFORMATIONS AND MINIMAL SURFACES

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Joint work with **José Ginés Espín Buendía, Antonio Linero Bas ,and Daniel Peralta Salas**.

An *interval exchange transformation of n-interval*, abbreviately *n-IET*, is an injective map $T : D \subset [0,1] \to [0,1]$ such that:

• D is the union of n pairwise disjoint open intervals, $D = \cup_{i=1}^{n} I_i$, with $I_i =]a_i, a_{i+1}[$, $a_1 = 0$, $a_{n+1} = 1$ and $n \geq 2$;

• $T|_{I_i}$ is a map of constant slope equals to 1 or -1;

If T *reverses the orientation* in the interval set $\mathcal{F} = \{I_{f_1}, I_{f_2}, \ldots, I_{f_k}\}$ (the slope is -1 in these intervals) for some $1 \leq f_j \leq n$ then we stress it by saying that T is *an interval exchange transformation of n-intervals with k-flips* or *an (n,k)-IET*; otherwise we say that T is *an interval exchange transformation of n-intervals without flips* or *an oriented interval exchange transformation of n-intervals.*

The point a_i is said to be a *false discontinuity* if $\lim_{x\to a_i^+} T(x) = \lim_{x\to a_i^-} T(x)$. We will say that T is a *proper* (n,k)*-IET* when it has not false discontinuities.

Let $x \in [0,1]$ then the orbit of x under T is the set:

$$\mathcal{O}_T(x) = \{T^n(x) : n \text{ is an integer and } T^n(x) \text{ makes sense}\}.$$

T is said to be *minimal* if for any $x \in [0,1]$ then $\mathcal{O}_T(x)$ is dense in $[0,1]$.

Let μ denote the standard Lebesgue measure on $[0,1]$. It is easy to see that μ (and any of its multiples) is an invariant measure for any interval exchange transformation. T will be said to be *uniquely ergodic* if it does not admit other invariant measures. We will present some progress in the theory of interval exchange transformations with flips. In particular we will pay attention to the Main Theorem in [2] which assures the existence of minimal, uniquely ergodic, proper (n,k)-IET's for any $n,k \in \mathbb{N}$ with $n \geq 4$ and $1 \leq k \leq n$. Also it will be explained the study non-oriented surfaces admitting minimal flows made in [1] and the work in progress to build minimal non uniquely ergodic flipped IET's.

Acknowledgements

This talk has been partially supported by the grant number MTM 2017-84079-P from Ministerio de Ciencia Innovación y Universidades (Spain)

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BERNOULLI DISJOINTNESS

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Joint work with **Eli Glasner, Todor Tsankov, and Benjamin Weiss**.

We consider the concept of disjointness for topological dynamical systems, introduced by Furstenberg. We show that for every discrete group, every minimal flow is disjoint from the Bernoulli shift. We apply this to give a negative answer to the Ellis problem for all such groups. For countable groups, we show in addition that there exists a continuum-sized family of mutually disjoint free minimal systems. Using this, we can identify the underlying space of the universal minimal flow of every countable group, generalizing results of Balcar-Blaszczyk and Turek. In the course of the proof, we also show that every countable ICC group admits a free minimal proximal flow, answering a question of Frisch, Tamuz, and Vahidi Ferdowsi.