DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D13

AGH University of Science and Technology

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GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D1

THE FRACTIONAL SUSCEPTIBILITY FUNCTION FOR THE QUADRATIC FAMILY

Viviane Baladi
CNRS & Sorbonne Université, France
Joint work with **Daniel Smania**.

For t in a set Ω of positive measure, maps in the quadratic family $f_t(x) = t - x^2$ admit an SRB measure μ_t. On the one hand, the dependence of μ_t on t has been shown [1] to be no better than $1/2$ Hölder, on a subset of Ω, for t_0 a suitable Misiurewicz-Thurston parameter. On the other hand, the susceptibility function $\Psi_t(z)$, whose value at $z = 1$ is a candidate for the derivative of μ_t with respect to t, has been shown [2] to admit a holomorphic extension at $z = 1$ for $t = t_0$. Our goal is to resolve this paradox. For this, we introduce and study a fractional susceptibility function.

References

[1] V. Baladi, M. Benedicks, and D. Schnellmann, *Whitney Hölder continuity of the SRB measure for transversal families of smooth unimodal maps*, Invent. Math. **201** (2015), 773-844.

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UNIQUE ERGODICITY FOR FOLIATIONS ON COMPACT KAEHLER SURFACES

Tien-Cuong Dinh
National University of Singapore, Singapore
Joint work with **Viet-Anh Nguyen and Nessim Sibony**.

Let F be a holomorphic foliation by Riemann surfaces on a compact Kaehler surface. Assume it is generic in the sense that all the singularities are hyperbolic and that the foliation admits no directed positive closed $(1,1)$-current, or equivalently, no invariant measure. Then there exists a unique (up to a multiplicative constant) positive ddc-closed $(1,1)$-current directed by F, or equivalently, a unique harmonic measure. This is a very strong ergodic property showing that all leaves of F have the same asymptotic behavior. Our proof uses an extension of the theory of densities to a new class of currents. A complete description of the cone of directed positive ddc-closed $(1,1)$-currents (i.e. harmonic measures) is also given when F admits directed positive closed currents (i.e. invariant measures).

MEASURE RIGIDITY FOR HIGHER RANK DIAGONALIZABLE ACTIONS

Manfred Einsiedler
ETH Zürich, Switzerland
Joint work with **Elon Lindenstrauss**.

We review old and recent measure rigidity results for higher rank diagonalizable actions on homogeneous spaces and contrast these results with the rank one and unipotent case. After this we consider higher rank actions on irreducible arithmetic quotients of $\mathrm{SL}_2(\mathbb{R})^k$ for $k \geq 2$. If the quotient is compact, positive entropy of an ergodic invariant measure μ implies algebraicity of μ with semisimple stabiliser. For non-compact quotients there are more possibilities. The main novelty here is that the acting group does not have to be maximal or in a special position. The main new idea is to use a quantitative recurrence phenomenon to transport positivity of entropy for one acting element to another.

EQUIDISTRIBUTION FOR COMMUTING MAPS

Michael Hochman
Hebrew University of Jerusalem, Israel

In two classical papers circa 1960, J. Cassels and W. Schmidt proved that a.e. numbers in the ternary Cantor set (with respect to Cantor-Lebesgue measure) eqidistributes for Lebesgue measure under the map $Tx = bx \bmod 1$, whenever b is an integer that is not a power of 3. This phenomenon has since been established in much greater generality on the interval, e.g. Host's theorem, according to which one can replace Cantor-Lebesgue measure by any $\times 3$-ergodic measure of positive entropy, provided $\gcd(3, b) = 1$. In this talk I will describe a new and heuristically simple proof of such results, and then discuss how it can be extended to give new results in multi-dimensional settings.

ON DYNAMICAL SPECTRAL RIGIDITY OF PLANAR DOMAINS

Vadim Kaloshin
University of Maryland, College Park, USA

Consider a convex domain on the plane and the associated billiard inside. The length spectrum is the closure of the union of perimeters of all period orbits. The length spectrum is closely related to the Laplace spectrum, through so-called the wave trace. The well-known question popularized by M. Kac: "Can you hear the shape of a drum?" asks if the Laplace spectrum determines a domain up to isometry. We call a domain dynamically spectrally rigid (DSR) if any smooth deformation preserving the length spectrum is an isometry. During the talk I will discuss recent results on DSR of convex planar domains.

ON THE DIVERGENCE OF BIRKHOFF NORMAL FORMS

Raphaël Krikorian
CNRS & Université de Cergy-Pontoise, France

A real analytic hamiltonian or a real analytic exact symplectic diffeomorphism admitting a non resonant elliptic fixed point is always formally conjugated to a formal integrable system, its Birkhoff Normal Form (BNF). Siegel proved in 1954 that the formal conjugation reducing a hamiltonian to its BNF is in general divergent and Hakan Eliasson has asked whether the BNF itself could be divergent. Perez-Marco proved in 2001 that for any fixed non resonant frequency vector the following dichotomy holds: either any real analytic hamiltonian system admitting this frequency vector at the origin has a convergent BNF or for a prevalent set of hamiltonians admitting this frequency vector the BNF generically diverges. It is possible to exhibit examples of hamiltonian systems with diverging BNF (X. Gong 2012 or the recent examples of B. Fayad in 4 degrees of freedom). The aim of this talk is to give a complete answer to the question of the divergence of the BNF (in the setting of exact symplectic diffeomorphisms): for any non resonant frequency vector, the BNF of a real analytic exact symplectic diffeomorphism admitting this frequency vector at the origin, is in general divergent. This theorem is the consequence of the remarkable fact that the convergence of the formal object that is the BNF has dynamical consequences, in particular an abnormal abundance of invariant tori.

KINETIC THEORY FOR THE LOW-DENSITY LORENTZ GAS

Jens Marklof
University of Bristol, UK
Joint work with **Andreas Strombergsson**.

The Lorentz gas is one of the simplest and most widely-studied models for particle transport in matter. It describes a cloud of non-interacting gas particles in an infinitely extended array of identical spherical scatterers, whose radii are small compared to their mean separation. The model was introduced by Lorentz in 1905 who, following the pioneering ideas of Maxwell

and Boltzmann, postulated that its macroscopic transport properties should be governed by a linear Boltzmann equation. A rigorous derivation of the linear Boltzmann equation from the underlying particle dynamics was given, for random scatterer configurations, in three seminal papers by Gallavotti, Spohn and Boldrighini-Bunimovich-Sinai. The objective of this lecture is to develop an approach for a large class of deterministic scatterer configurations, including various types of quasicrystals. We prove the convergence of the particle dynamics to transport processes that are in general (depending on the scatterer configuration) not described by the linear Boltzmann equation. This was previously understood only in the case of the periodic Lorentz gas through work of Caglioti-Golse and Marklof-Strombergsson. Our results extend beyond the classical Lorentz gas with hard sphere scatterers, and in particular hold for general classes of spherically symmetric finite-range potentials. We employ a rescaling technique that randomises the point configuration given by the scatterers' centers. The limiting transport process is then expressed in terms of a point process that arises as the limit of the randomised point configuration under a certain volume-preserving one-parameter linear group action.

INFLECTION POINTS FOR LYAPUNOV SPECTRA

Mark Pollicott
University of Warwick, UK
Joint work with **Oliver Jenkinson and Polina Vytnova**.

The Lyapunov spectra for a dynamical system describes the size (Hausdorff dimension) of the set of points which have a given Lyapunov exponent. H. Weiss conjectured that the associated graph is convex, but Iommi and Kiwi constructed a simple counter example. We explore this problem further, constructing examples with any given number of points of inflection.

MANDELBROT SET SEEN BY HARMONIC MEASURE: THE SIMILARITY MAP

Grzegorz Świątek
Warsaw University of Technology, Poland
Joint work with **Jacek Graczyk**.

We study conformal quantities at generic parameters with respect to the harmonic measure on the boundary of the connectedness loci $\mathcal{M}_d$ for unicritical polynomials $f_c(z) = z^d + c$. It is known that these parameters are structurally unstable and have stochastic dynamics. In [3] it was shown that for c from a set of full harmonic measure in $\partial\mathcal{M}_d$ there exists a quasi-conformal similarity map Υ_c between phase and parameter spaces which is conformal at c. In a recent work [2] we prove $C^{1+\frac{\alpha}{d}-\epsilon}$-conformality, $\alpha = \mathrm{HD}(\mathcal{J}_c)$, of $\Upsilon_c(z) : \mathbb{C} \mapsto \mathbb{C}$ at typical $c \in \partial\mathcal{M}_d$ and establish that globally quasiconformal similarity maps $\Upsilon_c(z)$, $c \in \partial\mathcal{M}_d$, are C^1-conformal along external rays landing at c in $\mathbb{C} \setminus \mathcal{J}_c$ mapping onto the corresponding rays of $\mathcal{M}_d$. This conformal equivalence leads to a proof that the z-derivative of the similarity map $\Upsilon_c(z)$ at typical $c \in \partial\mathcal{M}_d$ is equal to $1/\mathcal{T}'(c)$, where

$$\mathcal{T}(c) = \sum_{n=0}^{\infty} \left(D_z \left[f_c^n(z)\right]_{z=c}\right)^{-1}$$

is the transversality function previously studied by Benedicks-Carleson and Levin, see [1, 4]. There are additonal geometric consequences of these results. A typical external radius of the connectedness locus is contained in an asymptotically very nearly linear twisted angle, but nevertheless passes through infinitely many increasingly narrow straits.

References

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[2] J. Graczyk, G. Świątek, *Analytic structures and harmonic measure at bifurcation locus*, arXiv 1904.09434 (2019).

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TALKS OF SESSION D13

ON MIXING PROPERTIES OF INFINITE MEASURE PRESERVING TRANSFORMATIONS

Jon Aaronson
Tel Aviv University, Israel

I'll review the Hopf-Krickeberg mixing property with examples and discuss related ergodic properties such as rational weak mixing.

RECENT PROGRESS ON STRUCTURE AND CLASSIFICATION IN ERGODIC THEORY

Tim Austin
University of California, Los Angeles, USA

A basic formulation of the 'classification problem' asks for criteria to determine when two ergodic measure-preserving systems are isomorphic. It goes back to the foundational work of von Neuman and Halmos.

In more recent decades, this problem has evolved from 'pure' ergodic theory into a point of overlap with descriptive set theory. Culminating in work of Foreman, Rudolph and Weiss, this connection has shown the impossibility of any reasonable such classification. More recently, Foreman and Weiss have also shown that the important restriction of the classification problem to 'classical systems' - that is, smooth, volume-preserving maps of compact manifolds - is equally intractable.

However, some rich positive results are available in the direction of partial structure. These have the flavour that, for all ergodic measure-preserving systems, some 'soft' feature can be turned into a factor map or isomorphism to another system of a special kind. For instance, rotations on compact Abelian groups account for any failure of weak mixing (Halmos-von Neumann), and positive entropy can be fully realized by a Bernoulli-shift factor (Sinai). The second of these results was recently strengthened to show that all ergodic systems have Thouvenot's weak Pinsker property: they can always be split as a direct product of (i) a system with arbitrarily little entropy and (ii) a Bernoulli shift.

I will give a rough overview of some recent developments in this area and of some related settings in which many questions remains open. For the latter, I will especially emphasize the world of measure-preserving actions of sofic, non-amenable groups such as free groups.

EXPLICIT RESONANCES FOR ANALYTIC HYPERBOLIC MAPS

Oscar Bandtlow
Queen Mary University of London, UK
Joint work with **Wolfram Just and Julia Slipantschuk**.

In a seminal paper Ruelle showed that the long-time asymptotic behaviour of analytic hyperbolic systems can be understood in terms of the eigenvalues, also known as Pollicott-Ruelle resonances, of the so-called Ruelle transfer operator, a compact operator acting on a suitable Banach space of holomorphic functions.

In this talk I will report on recent work with Wolfram Just and Julia Slipantschuk on how to construct analytic expanding circle maps or analytic Anosov diffeomorphisms on the torus with explicitly computable non-trivial Pollicott-Ruelle resonances. I will also discuss applications of these results.

GENERIC BIRKHOFF SPECTRA

Zoltán Buczolich
Eötvös Loránd University, Hungary
Joint work with **Balázs Maga and Ryo Moore**.

Suppose that $\Omega = \{0,1\}^{\mathbb{N}}$ and σ is the one-sided shift. The Birkhoff spectrum $S_f(\alpha) = \dim_H \left\{\omega \in \Omega : \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(\sigma^n \omega) = \alpha\right\}$, where $\dim_H$ is the Hausdorff dimension. It is well-known that the support of $S_f(\alpha)$ is a bounded and closed interval $L_f = [\alpha^*_{f,\min}, \alpha^*_{f,\max}]$ and $S_f(\alpha)$ on L_f is concave and upper semicontinuous. We are interested in possible shapes/properties of the spectrum, especially for generic/typical $f \in C(\Omega)$ in the sense of Baire category. For a dense set in $C(\Omega)$ the spectrum is not continuous on $\mathbb{R}$, though for the generic $f \in C(\Omega)$ the spectrum is continuous on $\mathbb{R}$, but has infinite one-sided derivatives at the endpoints of L_f. We give an example of a function which has continuous S_f on $\mathbb{R}$, but with finite one-sided derivatives at the endpoints of L_f. The spectrum of this function can be as close as possible to a "minimal spectrum". We use that if two functions f and g are close in $C(\Omega)$ then S_f and S_g are close on L_f apart from neighborhoods of the endpoints.

SYMBOLIC EXTENSIONS AND UNIFORM GENERATORS FOR TOPOLOGICAL REGULAR FLOWS

David Burguet
CNRS & Sorbonne Université, France

Building on the theory of symbolic extensions and uniform generators for discrete transformations we develop a similar theory for topological regular flows. In this context a symbolic extension is given by a suspension flow over a subshift.

References

[1] D. Burguet, *Symbolic extensions and uniform generators for topological regular flows*, Journal of Differential Equations, (to appear), https://arxiv.org/abs/1812.04285.

SOME RESULTS ON PREDICTIVE SEQUENCES

Nishant Chandgotia
Hebrew University of Jerusalem, Israel
Joint work with **Benjamin Weiss**.

A sequence of natural numbers P is called *predictive* if for any zero-entropy stationary process X_i, X_0 is measurable with respect to X_{-i}; $i \in P$. In this talk, we will discuss several necessary conditions and sufficient conditions for sequences to be predictive.

GENERIC NONSINGULAR POISSON SUSPENSION IS OF TYPE III_1

Alexandre Danilenko
Institute for Low Temperature Physics and Engineering, NAS, Ukraine
Joint work with **Emmanuel Roy and Zemer Kosloff**.

Let (X, μ) be a standard measure space equipped with a non-atomic σ-finite infinite measure and let $\mathrm{Aut}(X, \mu)$ denote the group of all μ-nonsingular transformations of X. The Poisson suspension (X^*, μ^*) of (X, μ) is a well defined Lebesgue space. Then

$$\mathrm{Aut}_2(X,\mu) := \left\{ T \in \mathrm{Aut}(X,\mu) \mid \sqrt{\frac{d\mu \circ T}{d\mu}} - 1 \in L^2(X,\mu) \right\}$$

is the largest subgroup of $\mathrm{Aut}(X, \mu)$ consisting of those T for which the Poisson suspension T_* is μ^*-nonsingular [1]. It contains strictly the group

$$\mathrm{Aut}_1(X,\mu) := \left\{ T \in \mathrm{Aut}(X,\mu) \mid \frac{d\mu \circ T}{d\mu} - 1 \in L^1(X,\mu) \right\}$$

introduced in [2].

$\mathrm{Aut}_j(X, \mu)$ admits a natural Polish topology d_j stronger than the weak topology, $j = 1, 2$, and d_1 is stronger then d_2 [1]. There is a continuous homomorphism $\chi : \mathrm{Aut}_1(X, \mu) \to \mathbb{R}$, $\chi(T) := \int_X (\frac{d\mu \circ T}{d\mu} - 1) d\mu$ [1, 2].

Theorem 1. $\{T \in \mathrm{Aut}_2(X, \mu) \mid T$ is ergodic of type III_1 and T_* is ergodic of type $III_1\}$ is a dense G_δ in d_2.

Theorem 2. $\{T \in \mathrm{Ker}\chi \mid T$ is ergodic of type III_1 and T_* is ergodic of type $III_1\}$ is a dense G_δ in $(\mathrm{Ker}\ \chi, d_1)$.

Theorem 3. If $T \in \mathrm{Aut}_1(X, \mu)$ and $\chi(T) \neq 0$ then T_* is totally dissipative.

Example. There is a totally dissipative $T \in \mathrm{Aut}_1(X, \mu)$ such that for each real $t \in (0, \frac{1}{4})$, the Poisson suspension $(X^*, (t\mu)^*, T_*)$ is conservative but for each $t > 2$, the Poisson suspension $(X^*, (t\mu)^*, T_*)$ is totally dissipative.

References

[1] A.I. Danilenko, E. Roy, Z. Kosloff, *Nonsingular Poisson suspensions*, in preparation.

[2] Yu A. Neretin, *Categories of symmetries and infinite-dimensional groups*, Oxford University Press, 1996.

SARNAK CONJECTURE IN DENSITY

Thierry de la Rue
CNRS & Université de Rouen Normandie, France
Joint work with **Alexander Gomilko and Mariusz Lemańczyk**.

The talk will be based on a recent joint work with Alexander Gomilko and Mariusz Lemańczyk, in which we establish the following result related to Sarnak conjecture. (Here μ denotes the classical Möbius arithmetic function.)

If (X,T) is a zero entropy topological dynamical system with at most countably many invariant measures, then there exists a subset $A = A(X,T)$ of full logarithmic density in the set of natural integers, such that for any f continuous on X,

$$\sup_x \frac{1}{N} \sum_{1\le n\le N} \mu(n) f(T^n x) \longrightarrow 0, \quad \text{as } N \to \infty,\ N \in A.$$

The main tools are the results about logarithmic Furstenberg systems of the Möbius function proved by Frantzikinakis and Host [1], the logarithmic version of the so-called strong MOMO property [2], and an argument inspired by Tao to pass from logarithmic averages to classical averages along a subsequence of full logarithmic density.

References

[1] N. Frantzikinakis, B. Host, *The logarithmic Sarnak conjecture for ergodic weights*, Annals Math. **187** (2018), 869–931.

[2] E. H. El Abdalaoui, J. Kułaga-Przymus, M. Lemańczyk, T. de la Rue, *Möbius disjointness for models of an ergodic system and beyond*, Israel J. Math. **228** (2018), 707–751.

A CRUSH COURSE ON SYMBOLIC EXTENSIONS OF $\mathbb{Z}$-ACTIONS

Tomasz Downarowicz
Wrocław University of Science and Technology, Poland

Given a dynamical system (X, T), where T is a homeomorphism of a compact metric space X, we seek for its *lossless digitalization* in form of a subshift (Y, σ), where $Y \subset \Lambda^{\mathbb{Z}}$ (Λ is a finite alphabet) and σ denotes the standard shift, such that (X, T) is a topological factor of (Y, σ). It is obvious that a symbolic extension exists only for systems with finite topological entropy. But this condition is not sufficient. It turns out that in order to decide which systems admit symbolic extensions and how small can be their entropy one needs to study subtle entropy properties captured by the so-called entropy structure. In my talk I will try to present the most crucial definitions and facts around this topic.

References

[1] M. Boyle, T. Downarowicz, *The entropy theory of symbolic extensions*, Invent. Math. **156** (2004), 119-161.

[2] T. Downarowicz, *Entropy in Dynamical Systems*, Cambridge University Press, Cambridge, 2011.

$\mathcal{B}$-FREE NUMBERS FROM DYNAMICAL POINT OF VIEW

Aurelia Dymek

Nicolaus Copernicus University in Toruń, Poland

Joint work with **Stanisław Kasjan, Joanna Kułaga-Przymus and Mariusz Lemańczyk**.

Let $\mathcal{B} \subset \{2, 3, \ldots\}$. We call an integer n a *$\mathcal{B}$-free number* if n has no factor in $\mathcal{B}$. We denote the set of all $\mathcal{B}$-free integers by $\mathcal{F}_{\mathcal{B}}$. We consider the characteristic function of $\mathcal{F}_{\mathcal{B}}$ in the space of binary sequences and denote it by η. The subshift given by the orbit closure of η is called $\mathcal{B}$-free system and denoted by X_η. A prominent example of a such system is the square-free system which is studied since 2010 [2]. In this case the frequencies of blocks yields a natural shift-invariant ergodic measure on $\{0, 1\}^{\mathbb{Z}}$. It is called the *Mirsky measure*.

During the talk I will concentrate on same ergodic properties of $\mathcal{B}$-free systems (genericity, entropy, invariant measures) and give some combinatorial applications [1].

References

[1] A. Dymek, S. Kasjan, J. Kułaga-Przymus, M. Lemańczyk, *$\mathcal{B}$-free sets and dynamics*, Trans. Amer. Math. Soc. **370(8)** (2018), 5425–5489.

[2] P. Sarnak, *Three lectures on the Möbius function, randomness and dynamics*, http://publications.ias.edu/sarnak/.

INDUCING SCHEMES FOR PIECEWISE EXPANDING MAPS OF $\mathbb{R}^n$

Peyman Eslami
University of Rome Tor Vergata, Italy

For piecewise expanding maps of $\mathbb{R}^n$ I will show how to construct an inducing scheme where the base map is Gibbs-Markov and the return times have exponential tails. The existence of such a structure has many consequences in regards to the statistical properties of systems with discontinuities and non-uniform expansion.

PROGRESS ON THE SMOOTH REALIZATION PROBLEM

Matthew D. Foreman
University of California, Irvine, USA
Joint work with **Benjamin Weiss**.

We discuss a Global Structure Theorem for measure preserving transformations that has two corollaries:

1. For all Choquet simplices $\mathcal{K}$ there is an ergodic Lebesgue-measure preserving diffeomorphism of the 2-torus that has $\mathcal{K}$ as its simplex of invariant measures.

2. For all countable ordinals α there is a measure distal, measure preserving diffeomorphism of the 2-torus that has distal height α.

The first result changes Toeplitz systems built by Downarowicz into transformations that can be realized as diffeomorphisms. The second result stands in contrast to work of Rees, who showed that in the category of topological distality, the distal height is bounded by the dimension of the manifold.

References

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[4] M. Rees, *On the structure of minimal distal transformation groups with topological manifolds as phase spaces*, Ph.D. Thesis, University of Warwick (1977).

[5] M. Foreman and B. Weiss, *A Symbolic Representation of Anosov-Katok systems*, Journal d'Analyse Mathématique **137** (2019), 603-661.

[6] M. Foreman and B. Weiss, *From Odometers to Circular Systems: A Global Structure Theorem*, ArXiv 1703.07093, March (2017).

MULTIFRACTAL ANALYSIS FOR PLANAR SELF-AFFINE SETS

Thomas M. Jordan
University of Bristol, UK
Joint work with **Balázs Bárány, Antti Käenmäki, and Michał Rams**.

There is a standard problem in multifractal analysis of looking at level sets determined by the Birkhoff average of a suitable function. We look at the problem for self-affine sets on the plane. We show how recent work by Bárány, Hochman and Rapaport combined with results on approximation of pressure functions on suitable subsystems can give fairly complete solutions to this problem under certain generic algebraic assumptions and suitable separation assumptions.

THE NUMBER OF ERGODIC INVARIANT MEASURES FOR BRATTELI DIAGRAMS

Olena Karpel
AGH University of Science and Technology, Poland & Institute for Low Temperature Physics and Engineering, NAS, Ukraine
Joint work with **Sergey Bezuglyi and Jan Kwiatkowski**.

We study the simplex $\mathcal{M}_1(B)$ of probability measures on a Bratteli diagram B which are invariant with respect to the tail equivalence relation. Equivalently, $\mathcal{M}_1(B)$ is formed by probability measures invariant with respect to a homeomorphism of a Cantor set. We prove a criterion of unique ergodicity of a Bratteli diagram. In the case of a finite rank k Bratteli diagram B, we give a criterion for B to have exactly $1 \leq l \leq k$ ergodic invariant measures and describe the structures of the diagram and the subdiagrams which support these measures. We also find sufficient conditions under which a Bratteli diagram of arbitrary rank has a prescribed number (finite or infinite) of probability ergodic invariant measures.

PERIODS AND FACTORS OF WEAK MODEL SETS

Gerhard Keller
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
Joint work with **Christoph Richard**.

Weak model sets are defined by a cut-and-project scheme (CPS) $(G, H, \mathcal{L}, W)$, where G and H are locally compact second countable abelian groups, $\mathcal{L} \subset G \times H$ is a cocompact lattice, and $W \subset H$ is a compact set called the *window*. Denote by π_G and π_H the canonical projections from $G \times H$ to G and H, respectively. It is assumed that $\pi_G|_{\mathcal{L}}$ is $1-1$ and that $\pi_H(\mathcal{L})$ is dense in H. Typical cases to think of are $G = \mathbb{Z}^d$ or $\mathbb{R}^d$, while H could be $\mathbb{R}^k$ or an odometer group.

Denote $\mathbb{T} := (G \times H)/\mathcal{L}$. For $t \in \mathbb{T}$ the set $\Lambda_t := \pi_G((G \times W) \cap (t + \mathcal{L})) \subset G$ is a *weak model set*. The structure of Λ_t can be studied (besides many other possibilities) using dynamical systems methods: To that end define $X := \overline{\{\Lambda_t : t \in \mathbb{T}\}}$ and its subset $X_0 := \overline{\{\Lambda_0 + g : g \in G\}}$, where the topology stems from the vague topology on the space of locally finite measures on G when $\Lambda \subset G$ is identified with the *Dirac comb* $\sum_{x \in \Lambda} \delta_x$. In this way both spaces are compact metrizable, and G acts on them by translation.

Model sets, i.e. the case when $\overline{\mathrm{int}(W)} = W$, were originally studied by Y. Meyer [4], motivated by problems in harmonic analysis. There dynamical aspects are much studied and well understood. If W is aperiodic ($W + h = W \Rightarrow h = 0$, always true if $H = \mathbb{R}^d$), then (X, G) is an almost 1-1 extension of its maximal equicontinuous factor (MEF) $(\mathbb{T}, G)$, and if also $|\partial W| = 0$, then Haar-a.e. fibre of this factor map is a singleton. Examples are Sturmian sequences, Toeplitz sequences, the set of vertices of a Penrose tiling, and many others. See [5, 1] for reference.

But also the situation when $\overline{\mathrm{int}(W)}$ is a strict subset of W is of considerable interest; it suffices to mention the set of square free integers or the visible lattice points, which are weak model sets with compact groups H and $\mathrm{int}(W) = \emptyset$. Sets of $\mathcal{B}$-free numbers provide many other, intermediate examples. In [2] we prove among others:

Theorem A. $(\mathbb{T}/_{\mathbb{H}_{\mathrm{int}(W)}}, G)$ is the MEF of (X, G), where $\mathbb{H}_{\mathrm{int}(W)} = \{(0, h) \in G \times H : \mathrm{int}(W) + h = \mathrm{int}(W)\}$. If $\mathbb{H}_W = \mathbb{H}_{\mathrm{int}(W)}$, this is an almost 1-1 extension.

Remark. If $\mathrm{int}(W) = \emptyset$, the MEF is thus trivial. But if W is aperiodic and Haar regular, the *maximal equicontinuous generic factor* is still $(\mathbb{T}, G)$, see [3].

Theorem B. (X, G, Q) is measure theoretically isomorphic to $(\mathbb{T}/_{\mathbb{H}_W^{Haar}}, G, |\,.\,|)$, where $|\,.\,|$ denotes Haar measure, $\mathbb{H}_W^{Haar} = \{(0, h) \in G \times H : |(W + h) \triangle W| = 0\}$, and Q is the image of the Haar measure on $\mathbb{T}$ under the map $t \mapsto \Lambda_t$ (called *Mirsky measure* in arithmetic contexts).

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AUTOMATIC SEQUENCES, NILSYSTEMS, AND HIGHER ORDER FOURIER ANALYSIS

Jakub Konieczny
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Joint work with **Jakub Byszewski and Clemens Müllner**.

Automatic sequences give rise to one of the basic models of computation and have remarkable links to many areas of mathematics, including dynamics, algebra and logic. Distribution of these sequences has long been studied. During the talk we will explore this topic from the point of view of higher order Fourier analysis. As it turns out, many of the classical automatic sequences are highly Gowers uniform, while others can be expressed as the sum of a structured component and a uniform component much more efficiently than guaranteed by the arithmetic regularity lemma. We investigate the extent to which this phenomenon extends to general automatic sequences and consider some closely related problems that make sense for sparse sequences.

MULTIPLICATIVE FUNCTIONS AND DISJOINTNESS IN ERGODIC THEORY

Mariusz Lemańczyk
Nicolaus Copernicus University in Toruń, Poland

In 2010, P. Sarnak formulated the Möbius orthogonality conjecture stating that the classical Möbius function does not correlate with any continuous observable in a (topological) zero entropy dynamical system. This conjecture has deep connections with analytic number theory and joinings in ergodic theory. My talk will be devoted to present some of these connections and an overview of the latest achievements.

TBA

Christian Mauduit
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A STRICTLY ERGODIC, POSITIVE ENTROPY SUBSHIFT UNIFORMLY UNCORRELATED TO THE MÖBIUS FUNCTION

Jacek Serafin
Wrocław University of Science and Technology, Poland
Joint work with **Tomasz Downarowicz**.

This talk is based on two recent papers [1] and [2], where we show that if $y = (y_n)_{n\geq 1}$ is a bounded sequence with zero average along every infinite arithmetic progression then for every $N \geq 2$ there exists a strictly ergodic subshift Σ over N symbols, with entropy arbitrarily close to $\log N$, uniformly uncorrelated to y. In particular, for $y = \mu$ being the Möbius function, there exist subshifts as above which satisfy the assertion of Sarnak's conjecture ([3]). To the best of our knowledge, no other examples of positive entropy systems uncorrelated to the Möbius sequence are known.

Our result shows, among other things, that (even for strictly ergodic systems) the so-called strong MOMO (Möbius Orthogonality on Moving Orbits) property is essentially stronger than uniform uncorrelation.

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INVARIANT MEASURES FOR RANDOM WALKS ON HOMEO$^+$(R)

Tomasz Szarek
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Joint work with **D. Buraczewski and S. Brofferio**.

Let g_n be a sequence of i.i.d. Homeo$^+$(R)–valued randomvariables whose distribution is a measure μ. We consider the left randomwalk on Homeo$^+$(R) defined by the random variables $f_n := g_n \circ \cdots \circ g_1$. We study the Markov chain (X_n) on the real line corresponding to g_n, i.e.for any $x \in \mathbb{R}$ and $n \in \mathbb{N}$ we consider the random variables defined by $X_n^x := f_n(x)$. The main purpose of the talk is to provide suffcient conditions forthe existence of a unique invariant Radon measure (mainly infinite) for(X_n). This research generalizes the results obtained by Deroin, Kleptsyn,Navas and Parvani, who studied similar problems for groups of homeomorphisms.

LIMIT PROPERTIES FOR WOBBLY INTERMITTENT MAPS

Dalia Terhesiu
Leiden University, Netherlands
Joint work with **Douglas Coates and Mark Holland**.

It is known that finite measure preserving intermittent maps with indifferent fixed points characterised by regular variation satisfy stable laws for sufficiently regular observables that do not vanish at the indifferent fixed points. We consider a finite measure preserving Pomeau Manneville type map, perturb the behaviour at the (only one) indifferent fixed point according to a St. Petersburg type distribution and obtain convergence to a non trivial limit distribution (a semistable law) along subsequences. Also, we obtain lower bounds on the decay of correlation for such modified maps and suitable observables. In this talk I will present these results.

COHOMOLOGICAL THEORY OF THE SEMI-CLASSICAL ZETA FUNCTIONS

Masato Tsujii
Kyushu University, Japan

We first review very briefly about recent developments in analysis of transfer operators for hyperbolic dynamical systems. We will then focus on the semi-classical (or Gutzwiller-Voros) zeta functions for geodesic flows on negatively curved manifolds. We show that the semi-classical zeta function is the dynamical Fredholm determinant of a transfer operator acting on the leaf-wise cohomology space along the unstable foliation. This realize the idea presented by Guillemin and Patterson a few decades ago. As an application, we see that the zeros of the semi-classical zeta function concentrate along the imaginary axis, imitating those of Selberg zeta function.

References

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TIME-CHANGES PRESERVING ZETA FUNCTIONS

Thomas Ward
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Joint work with **Sawian Jaidee and Patrick Moss**.

A time-change is a function $h\colon \mathbb{N} \to \mathbb{N}$, and h is said to 'preserve zeta functions' if, for any dynamical zeta function $\exp\left(\sum_{n\geq 1} a_n z^n/n\right)$, where $a_n = |\{x \in X \mid T^n x = x\}|$ for some dynamical system $T\colon X \to X$, the time-changed function $\exp\left(\sum_{n\geq 1} a_{h(n)} z^n/n\right)$ is the dynamical zeta function of some dynamical system. That is, for any homeomorphism of a compact metric space $T\colon X \to X$ there is some other homeomorphism of a compact metric space $S\colon Y \to Y$ with the property that $|\{x \in X \mid T^{h(n)} x = x\}| = |\{y \in Y \mid S^n y = y\}|$ for all $n \in \mathbb{N}$. The time-changes that preserve zeta functions form a monoid $\mathcal{P}$, and we show that a polynomial lies in $\mathcal{P}$ if and only if it is a monomial (meaning that $\mathcal{P}$ is algebraically small), that $\mathcal{P}$ is uncountable (meaning that it is set-theoretically large), and that $\mathcal{P}$ contains no permutations (that is, $\mathcal{P}$ has no torsion as a monoid).

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ON THE COMPLEXITY OF SMOOTH SYSTEMS

Benjamin Weiss
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Joint work with **Matthew Foreman**.

About ten years ago, in joint work with the late Dan Rudolph and Matt Foreman, we showed that the isomorphism relation for ergodic measure systems is not Borel, but rather a complete analytic set. In fact we showed that the transformations that are isomorphic to their inverses is already complete analytic. Since the smooth realization problem is still open it was not clear how complex is the class of diffeomorphisms of compact manifolds that preserve a volume element. In more recent work with Matt Foreman we show that already the ergodic diffeomorphisms of the torus that preserve Lebesgue measure is also a complete analytic set.

ASYMPTOTIC h-EXPANSIVENESS FOR AMENABLE GROUP ACTIONS

Guohua Zhang
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Joint work with **Tomasz Downarowicz**.

Asymptotic h-expansiveness for amenable group actions can be introduced respectively using topological conditional entropy in [1] and using entropy structure in [2]. In this talk we will show the equivalence of these two kinds of asymptotic h-expansiveness.

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