DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D22

AGH University of Science and Technology

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Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D2

DIFFERENTIABILITY OF THE FLOW FOR BV VECTOR FIELDS

Stefano Bianchini
SISSA, Italy

We show that the Regular Lagrangian Flow $X(t,y)$ generated by nearly incompressible BV vector fields admits a derivative $\nabla X(t,y)$ in the sense of measure. This matrix satisfies the ODE

$$\frac{d}{dt}\nabla X(t,y) = (D\mathbf{b}(t))_y \nabla X(t-,y)$$

where $(D\mathbf{b})_y$ is the disintegration of the measure $\int D\mathbf{b}(t)dt$ w.r.t. the trajectories $X(t,y)$.

ON THE LARGE TIME BEHAVIOR OF SOLUTIONS TO BIRTH AND SPREAD TYPE EQUATIONS

Yoshikazu Giga
University of Tokyo, Japan
Joint work with **Hiroyoshi Mitake, Takeshi Ohtsuka, and Hung V. Tran**.

We consider a level-set eikonal-curvature flow equation with an external force. Such a problem is considered as a model to describe an evolution of height of crystal surface by two-dimensional nucleation or possibly some class of growths by screw dislocations. For applications, it is important to estimate growth rate. Without an external source term the solution only spreads horizontally and does not grow vertically so the source term plays a key role for the growth.

Although the large time behavior of parabolic equations are well studied, the equations we study are degenerate parabolic equations where no diffusion effect exists in the normal to each level-set of a solution. Thus, very little is known even for growth rate. Our goal is to describe our recent progress on such type of problems. Ealier results are presented in the paper by H. Mitake, H.V. Tran and the lecturer published in SIAM Math. Anal. in 2016. A review paper is published in Proc. Int. Cong. of Math. in 2018.

In this talk, we first show the existence of asymptotic speed called growth rate. We also discuss asymptotic profile as well as estimates on growth rate.

THE TWO HYPERPLANE CONJECTURE

David Jerison
Massachusetts Institute of Technology, USA

I will introduce a conjecture that I call the *Two Hyperplane Conjecture*, saying that an isoperimetric surface that divides a convex body in half by volume is trapped between parallel hyperplanes. Emanuel Milman has shown that in its strongest, dimension-independent form, my conjecture implies the *Hyperplane Conjecture* of Kannan, Lovász and Simonovits in theoretical computer science, which says that the area of such an isoperimetric surface is comparable, by an absolute constant independent the convex body and its dimension, to the area of some hyperplane dividing the convex body in half. Their conjecture is closely related to several famous unsolved problems in high dimensional convex geometry. But unlike the hyperplane conjecture, the two-hyperplane conjecture has significance even in low dimensions.

I will relate the conjecture to qualitative and quantitative connectivity properties and regularity of area-minimizing surfaces, free boundaries and level sets of eigenfunctions, and report on work in progress with Guy David. The main theme of the talk is that the level sets of least energy solutions to scalar variational problems should be as simple as possible, but no simpler.

ON THE NONLINEAR STABILITY OF BLACK HOLES

Sergiu Klainerman
Princeton University, USA

Black holes are precise mathematical solutions of the Einstein field equations mainly represented by the famous two parameter Kerr family including, as a particular case, the Schwarzschild solution. To correspond to physical reality, i.e. to be more than mathematical artifacts, these solutions have to be stable under small perturbations. While there is today no doubt concerning the physical reality of black holes, based both on observational data and numerical simulations, an actual proof of stability remains a fundamental challenge of Mathematical and Geometric Analysis.

In my talk I will formulate the precise mathematical problem of the nonlinear stability of the Kerr family and describe the main results known so far. In the second part of the talk I will describe my recent result with J. Szeftel "Global Nonlinear Stability of Schwarzschild Spacetime under Polarized perturbations" - arXiv:1711.07597. The result establishes the full nonlinear stability of Schwarzschild spacetime under axially symmetric, polarized perturbations, i.e. stability of solutions of the Einstein vacuum equations for asymptotically flat $1+3$ dimensional Lorentzian metrics which admit a hyper-surface orthogonal space-like Killing vector-field with closed orbits. While building on the remarkable advances made in last 15 years on establishing quantitative linear stability, the paper introduces a series of new ideas among which we emphasize the *general covariant modulation* (GCM) procedure which allows us to construct, dynamically, the center of mass frame of the final state. The mass of the final state itself is tracked using the well known Hawking mass relative to a well adapted foliation itself connected to the center of mass frame. Our work here is the first to prove the nonlinear stability of Schwarzschild in a restricted class of nontrivial perturbations. To a large extent, the restriction to this class of perturbations is only needed to ensure that the final state of evolution is another Schwarzschild space.

ZERO SETS OF LAPLACE EIGENFUCNTIONS

Aleksandr Logunov
Princeton University, USA

We will discuss geometrical and analytic properties of zero sets of harmonic functions and eigenfunctions of the Laplace operator. For harmonic functions on the plane there is an interesting relation between local length of the zero set and the growth of harmonic functions. The larger the zero set is, the faster the growth of harmonic function should be and vice versa. Laplace eigenfunctions on two dimensional sphere are restrictions of homogeneous harmonic polynomials of three variables onto the sphere. Zero sets of such functions are unions of smooth curves with equiangular intersections. Topology of the zero set could be quite complicated, but the total length of the zero set of any spherical harmonic of degree n is comparable to n.

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EFFECTIVE BEHAVIOR OF RANDOM MEDIA

Felix Otto
Max Planck Institute for Mathematics in the Sciences, Germany

In engineering applications, heterogeneous media are often described in statistical terms. This partial knowledge is sufficient to determine the effective, i.e. large-scale behavior. This effective behavior may be inferred from the Representative Volume Element (RVE) method. I report on last years' progress on the quantitative understanding of what is called *stochastic homogenization of elliptic partial differential equations*: optimal error estimates of the RVE method, leading-order characterization of fluctuations, effective multipole expansions. Methods connect to elliptic regularity theory and to concentration of measure arguments.

IMPLICITLY CONSTITUTED FLUID FLOW MODELS: ANALYSIS AND APPROXIMATION

Endre Süli
University of Oxford, UK

Classical models describing the motion of Newtonian fluids, such as water, rely on the assumption that the Cauchy stress is a linear function of the symmetric part of the velocity gradient of the fluid. This assumption leads to the Navier-Stokes equations. It is known however that the framework of classical continuum mechanics, built upon a simple linear constitutive equation for the Cauchy stress, is too narrow to describe inelastic behavior of solid-like materials or viscoelastic properties of materials. Our starting point in this work is therefore a generalization of the classical framework of continuum mechanics, called the implicit constitutive theory, which was proposed recently in a series of papers by Rajagopal. The underlying principle of the implicit constitutive theory in the context of viscous flows is the following: instead of demanding that the Cauchy stress is an explicit (and, in particular, linear) function of the symmetric part of the velocity gradient, one may allow a nonlinear, implicit and not necessarily continuous relationship between these quantities. The resulting general theory therefore admits non-Newtonian fluid flow models with implicit and possibly discontinuous power-law-like rheology.

We develop the analysis of finite element approximations of implicit power-law-like models for viscous incompressible fluids. The Cauchy stress and the symmetric part of the velocity gradient in the class of models under consideration are related by a, possibly multi-valued graph. Using a variety of weak compactness techniques, we show that when the graph of the stress-strain relationship is maximal monotone a subsequence of the sequence of finite element solutions converges to a weak solution of the problem as the discretization parameter, measuring the granularity of the finite element triangulation, tends to zero. When the graph is nonmonotone, a subsequence of the sequence of finite element solutions is shown to converge to a gradient Young-measure solution of the problem. A key new technical tool in the analysis is a finite element counterpart of the Acerbi-Fusco Lipschitz truncation of Sobolev functions. The talk is based on a series of papers with Miroslav Bulíček and Josef Málek (Prague), Miles Caddick (Oxford), Lars Diening (Bielefeld), Christian Kreuzer (Dortmund), and ongoing research with Alexei Gazca-Orozco (Oxford) and Tabea Tscherpel (Aachen).

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GLOBAL ANALYSIS VIA MICROLOCAL TOOLS: FREDHOLM THEORY IN NON-ELLIPTIC SETTINGS

András Vasy
Stanford University, USA
Joint work with **Peter Hintz**.

In this lecture I will describe a microlocal framework for the Fredholm analysis of non-elliptic problems both on manifolds without boundary and manifolds with boundary, introduced in [8] and extended in [6] and various other works. Examples in which such a framework (or a similar framework) has recently been useful include wave propagation on black hole spacetimes, which is the key analytic ingredient for showing the stability of black holes [5, 4], analysis of the resolvent of the generator of the flow for dynamical systems [2], which is the key tool for the analysis of the Ruelle zeta function [1], Feynman propagators in quantum field theory [3, 9] and inverse problems, namely boundary rigidity and tensor tomography [7].

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THE VORTEX FILAMENT EQUATION, THE TALBOT EFFECT, AND NON-CIRCULAR JETS

Luis Vega
University of the Basque Country & BCAM, Spain
Joint work with **Valeria Banica and Francisco De La Hoz**.

We will propose the vortex filament equation as a possible toy model for turbulence, in particular because of its striking similarity to the dynamics of non-circular jets. This similarity implies the existence of some type of Talbot effect due to the interaction of non-linear waves that propagate along the filament. Another consequence of this interaction is the existence of a new class of multi-fractal sets that can be seen as a generalization of the graph of Riemann's non-differentiable function. Theoretical and numerical arguments about the transfer of energy will be also given.

POPULATION DYNAMICS AND CONTROL

Enrique Zuazua
DeustoTech, Bilbao, Basque Country & Universidad Autónoma de Madrid, Spain

Population dynamics is an old subject. Classical models in this field are written in terms of reaction-diffusion equations.

There is a wide literature concerning the dynamical properties of those systems. But much less is known from a control perspective. And control constitutes the ultimate proof of our understanding of a process.

This lecture will be devoted to present two recent results in this area. We first consider a bistable reaction-diffusion arising in the modelling of bilingual populations and then address models involving age structuring and spatial diffusion (of Lotka-McKendrick type).

As we shall see, both aspects require of an in depth understanding of the dynamics of the systems under consideration.

We shall present sharp results on our ability to steer the dynamics of those systems to a prescribed final configuration. Some open problems and future directions of research will also be presented.

This lecture is based on recent joint work in collaboration with D. Maity, C. Pouchol, E. Trélat, M. Tucsnak and J. Zhu.

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TALKS OF SESSION D22

OBSERVABILITY, CONTROL AND STABILIZATION OF WATER WAVES

Thomas Alazard
École Normale Supérieure Paris-Saclay, France

The main questions studied in this talk are the generation and the absorption of water waves in a wave tank. We want to understand which water waves can be generated by blowing above a localized portion of the free surface of a liquid. We also want to understand how to damp water waves when they reach the boundary of a numerical wave tank.

For $2D$ water waves in a rectangular tank, the fluid domain $\Omega(t)$ is of the form

$$\Omega(t) = \{ (x, y) : x \in [0, L], h \leq y \leq \eta(t, x) \},$$

where x (resp. y) is the horizontal (resp. vertical) space variable, L is the width of the tank, h its depth and η is the free surface elevation. The equations which dictate the motion are the incompressible Euler equations with free surface. This is a system of two nonlinear equations: the incompressible Euler equation for the velocity potential $\phi \colon \Omega \to \mathbb{R}$ (so that the velocity is $v = \nabla_{x,y}\phi$) and a kinematic equation for η which states that the free surface moves with the fluid. Zakharov discovered that η is conjugated to the trace $\psi(t, x) = \phi(t, x, \eta(t, x))$ of the velocity potential on the free surface: the equations have the hamiltonian form

$$\frac{\partial \eta}{\partial t} = \frac{\delta \mathcal{H}}{\delta \psi}, \quad \frac{\partial \psi}{\partial t} = -\frac{\delta \mathcal{H}}{\delta \eta} - P_{ext}, \tag{1}$$

where P_{ext} is an external pressure and $\mathcal{H}$ is the energy

$$\mathcal{H} = \frac{g}{2}\int_0^L \eta^2(t, x)\, dx + \int_0^L \left(\sqrt{1 + (\partial_x \eta(t, x))^2} - 1\right) dx + \frac{1}{2}\iint_{\Omega(t)} |\nabla_{x,y}\phi(t, x, y)|^2 \, dxdy.$$

• The first problem is the following : given a time $T > 0$, a final state $(\eta_{final}, \psi_{final})$ in some space of regular functions, a non empty interval $\omega = (a, b) \subset [0, L]$, is-it possible to find a function $P_{ext}(t, x)$ supported in $[0, T] \times \omega$ such that the solution to (1) with initial data $(\eta_{in}, \psi_{in}) = (0, 0)$ satisfies $(\eta, \psi)|_{t=T} = (\eta_{final}, \psi_{final})$? I will present a local controllability result obtained with Pietro Baldi and Daniel Han-Kwan ([1]).

• We then consider the stabilization problem. The goal here is to find a pressure law, relating P_{ext} to the unknown (η, ψ) and supported inside a small subset of $[0, L]$, such that $\mathcal{H}$ is decreasing and converges to zero. I will explain how to use the multiplier method of C. Morawetz and J.L. Lions to study this problem ([2, 3, 4]).

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ONE-SIDE BOUNDARY CONTROLLABILITY OF THE *P*-SYSTEM

Fabio Ancona

University of Padua, Italy

Joint work with **Olivier Glass and Khai T. Nguyen**.

We consider the equations for one-dimensional isentropic compressible gases on an interval, in Eulerian or in Lagrangian coordinates (known as the the p-system). On one side of the interval it is imposed a fixed boundary condition (for instance the null velocity), while on the other side of the interval the boundary condition is treated as a control that one can choose to influence the system. We prove a result of controllability toward constants states in the context of (discontinuous) weak entropy solutions. Namely, we prove that it is possible, starting from an initial state small in BV, to reach any constant state compatible with the boundary conditions. This type of result was previously obtained in [2] in the context of boundary controls acting on both boundaries. These results are in sharp contrast with what happens for some other 2×2 strictly hyperbolic systems with genuinely nonlinear characteristics fields for which Bressan and Coclite [1] showed that, in general, exact controllability to constant states is not possible, even when controlling on both sides of the interval.

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AN ERGODIC CONTROL PROBLEM WITH ROSENBLATT NOISE

Petr Čoupek
Charles University, Czech Republic
Joint work with **Tyrone E. Duncan, Bohdan Maslowski, and Bożenna Pasik-Duncan**.

The talk will be devoted to an infinite time horizon linear-quadratic control problem for a stochastic differential equation with additive Rosenblatt noise.

Rosenblatt processes arise naturally as limits of suitably normalized sums of long-range dependent random variables in a non-central limit theorem. These continuous processes are self-similar, have stationary increments and exhibit long memory; however, unlike the family of regular fractional Brownian motions, they are not Gaussian. This last property makes their analysis somewhat intriguing and it is also the reason why they received considerable attention in recent years.

Initially, some recent results on stochastic calculus for Rosenblatt and related fractional processes will be presented in the talk. Subsequently, the ergodic control problem will be formulated and solved; and the optimal control as well as the optimal cost will be given explicitly.

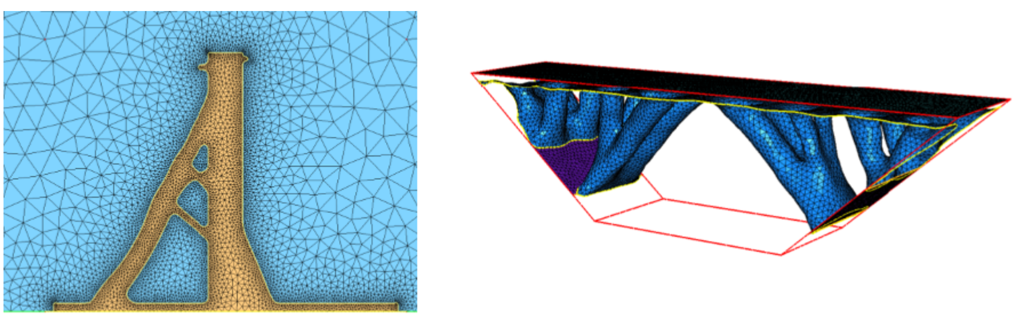
SHAPE AND TOPOLOGY OPTIMIZATION VIA A LEVEL-SET BASED MESH EVOLUTION METHOD

Charles Dapogny
Université Grenoble-Alpes, France
Joint work with **Grégoire Allaire, Florian Feppon, and Pascal Frey**.

The purpose of this presentation is to introduce a robust front-tracking method for dealing with arbitrary motions of shapes, even dramatic ones (e.g. featuring topological changes); although this method is illustrated in the particular context of shape optimization, it naturally applies to a wide range of inverse problems and reconstruction algorithms.

The presented method combines two different means of representing shapes: on the one hand, they are meshed explicitly, which allows for efficient mechanical calculations by means of any standard Finite Element solver; on the other hand, they are represented by means of the level set method, a format under which it is easy to track their evolution. The cornerstone of our method is a pair of efficient algorithms for switching from either of these representations to the other.

Several numerical examples are discussed in two and three space dimensions, in the 'classical' physical setting of linear elastic structures, but also in more involved situations involving e.g. fluid-structure interactions.



(Left) Optimal design of a mast withstanding an incoming flow; (right) optimal design of a bridge.

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A STACKELBERG STRATEGY TO CONTROL A HEAT EQUATION

Luz de Teresa
National Autonomous University of Mexico, Mexico
Joint work with **Enrique Fernández-Cara and José Antonio Villa**.

In this conference we present the control problem that arises with the application of multiple strategies to the control of parabolic equations. We assume that we can act on the system through a hierarchy of controls. The leader control has an optimization objective and the follower a null control objective. We discuss the differences that arise when the leader control has a null controllability objective while the follower an optimization one. Literature, as far as we know, treats the second problem, see [1, 2, 3, 4].

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HJB EQUATIONS IN INFINITE DIMENSION AND OPTIMAL CONTROL OF STOCHASTIC EVOLUTION EQUATIONS VIA GENERALIZED FUKUSHIMA DECOMPOSITION

Giorgio Fabbri
CNRS, France
Joint work with **Francesco Russo**.

In this talk we present the result of the paper [1].

We consider a stochastic optimal control problem driven by an abstract evolution equation in a separable Hilbert space. Thanks to the identification of the mild solution of the state equation as ν-weak Dirichlet process, the value processes is proved to be a real weak Dirichlet process. The uniqueness of the corresponding decomposition is used to prove a verification theorem.

Through that technique some of the required assumptions are milder than those employed in previous contributions about non-regular solutions of Hamilton-Jacobi-Bellman equations. We present some explicit example.

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OPTIMAL CONTROL OF STOCHASTIC EVOLUTION EQUATIONS VIA RANDOMIZATION AND BACKWARD SDES

Marco Fuhrman
University of Milan, Italy
Joint work with **Emanuela Gussetti**.

Backward Stochastic Differential Equations (BSDEs) have been successfully applied to represent the value of optimal control problems for controlled stochastic differential equations. Since in the classical framework several restrictions on the scope of applicability of this method remained, in recent times several approaches have been devised to obtain the desired probabilistic representation in more general situations.

We will review the so called randomization method, originally introduced by B. Bouchard in the framework of optimal switching problems, which consists in introducing an auxiliary, "randomized" problem with the same value as the original one, where the control process is replaced by an exogenous random point process, and optimization is performed over a family of equivalent probability measures. The value of the randomized problem is then represented by means of a special class of BSDEs with a constraint on one of the unknown processes.

This methodology will be applied in the framework of controlled evolution equations (with immediate applications to controlled SPDEs), a case for which very few results are known so far.

GLOBAL SOLUTIONS AND STABILITY FOR A 3-D FLUID-STRUCTURE INTERACTION

Irena Lasiecka
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We consider an interface problem consisting of a 3-D fluid equation interacting with a 3-D dynamic elasticity. The interface is moving according to the speed of the fluid. The PDE system is modeled by system of partial differential equations describing motion of an elastic body inside an incompressible fluid. The fluid is governed by Navier-Stokes equation while the structure is represented by the system of dynamic elasticity with weak dissipation. The interface between the two environments undergoes oscillations which lead to moving frame configuration, the latter giving rise to a quasilinear system. Short time local existence of solutions has been shown in [1]. The aim of the talk is to discuss global [in time] solutions under small disturbance hypothesis. Stability [in time] of such solutions is also considered along with some control problems related to minimization of the vorticity. The problem is motivated by applications arising in bio-mechanics, aeroelasticity and industrial processes. In the presence of weak damping affecting the solid the control-to-observation map is proved global-so that the size of the data can be chosen uniformly in time. This allows consideration of an infinite time horizon control problem. The latter depends on the global existence results obtained in [2].

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SHAPE DESIGN FOR SUPERCONDUCTORS GOVERNED BY H(CURL)-ELLIPTIC VARIATIONAL INEQUALITIES

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Joint work with **Malte Winckler and Irwin Yousept**.

We study a shape optimization problem governed by an elliptic curl-curl variational inequality (VI) of the second kind. We present a Moreau-Yosida type regularization for the dual formulation of the VI that guarantees the Gâteaux-differentiability of the regularized dual variable. Then, for a fixed regularization parameter, the existence of an optimal shape for the corresponding problem is proved by means of a compactness theorem. Then we analyze the sensitivity of the regularized objective functional by rigorously computing the corresponding shape derivative using the averaged adjoint method, a lagrangian-type formulation. We also give a stability estimate for the shape derivative with respect to the regularization parameter, and show the strong convergence of the optimal shapes and the corresponding states for the regularized problem towards a solution to the problem without regularization. Finally, we present the numerical algorithm based on the distributed shape derivative coupled with the level set method, and we apply it to problems stemming from the type-II (HTS) superconductivity.

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LOCAL CONTROLLABILITY TO THE TRAJECTORIES OF THE FOKKER-PLANCK EQUATION WITH A LOCALIZED CONTROL

Pierre Lissy
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Joint work with **Michel Duprez**.

We will present a new result on the control of the Fokker-Planck equation, posed on a smooth bounded domain of $\mathbb{R}^d$ ($d \geq 1$). More precisely, the control is the drift force, localized on a small open subset. We prove that this system is locally null controllable to regular nonzero trajectories, with potentially strictly less that d controls. The results are obtained thanks to a linearization method based on a standard inverse mapping procedure and the fictitious control method. The main novelties of the present article are twofold. Firstly, we propose an alternative strategy to the standard fictitious control method: the algebraic solvability is performed and used directly on the adjoint problem. Secondly, we prove a new Carleman inequality for the heat equation with one order space-varying coefficients: the right-hand side is the gradient of the solution localized on a subset (rather than the solution itself), and the left-hand side contains arbitrary high derivatives of the solution.

COMPETITION IN DEFENSIVE AND OFFENSIVE ADVERTISING STARTEGIES IN A SEGMENTED MARKET

Dominika Machowska
University of Łódź, Poland
Joint work with **Andrzej Nowakowski**.

We propose the new goodwill model à la Nerlove-Arrow defined on a competitive segmented market. Based on the dual dynamic approach, we give the sufficient condition under which the open-loop equilibrium exists for the new game. We also introduce ε-open loop equilibrium as a basis for the numerical algorithm using a construction of the optimal solution in the finite steps. The numerical algorithm enables an analysis of how the level of the homogeneity of given competitive products and customer recommendations modify optimal goodwill and the total profit of each player.

FILTERING AND OPTIMAL CONTROL AND FOR GAUSS-VOLTERRA PROCESSES IN HILBERT SPACES

Bohdan Maslowski
Charles University, Czech Republic
Joint work with **Tyrone E. Duncan, Bożenna Pasik-Duncan and Vít Kubelka**.

Kalman-Bucy type filter and some methods of parameter estimation are studied in the case when signals are Hilbert space-valued Gaussian processes. The corresponding integral equations are derived for the optimal estimate and covariance of the error. Some basic properties of the filter are discussed. These general results are illustrated by examples of linear SPDEs where the noise terms are Gauss-Volterra processes (in particular, fractional Brownian motions). Also, some optimal control results for such systems are recalled for the case of quadratic cost functionals. In both cases, the results are compared with the standard ones for Gauss-Markov systems.

ASYMPTOTIC SPHERICAL SHAPES IN SPECTRAL OPTIMIZATION PROBLEMS

Dario Mazzoleni
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Joint work with **Benedetta Pellacci and Gianmaria Verzini**.

We study the positive principal eigenvalue of a weighted problem associated with the Neumann-Laplacian settled in a box $\Omega \subset \mathbb{R}^N$, which arises from the investigation of the survival threshold in population dynamics. When trying to minimize such eigenvalue with respect to the sign-changing weight, one is lead to consider a shape optimization problem, which is known to admit spherical optimal shapes only in trivial cases. We investigate if spherical shapes can be recovered in the limit when the negative part of the weight diverges. First of all, we show that the shape optimization problem appearing in the limit is the so called *spectral drop* problem, which involves the minimization of the first eigenvalue of the mixed Dirichlet-Neumann Laplacian. Thanks to α-symmetrization techniques on cones, it can be proved that optimal shapes for the spectral drop problem are spherical for suitable choices of the box, the most interesting case being when Ω is a convex polytope, and in this case a quantitative analysis of the convergence can be performed. Finally, for a smooth Ω, we show that small volume spectral drops are asymptotically spherical, with center at points with high mean curvature.

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BOUNDARY CONTROLLABILITY OF A COUPLED FOURTH-SECOND ORDER PARABOLIC SYSTEM

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Joint work with **Nicolás Carreño and Eduardo Cerpa**.

We study a control system coupling fourth and second-order parabolic equations, when we only control the second-order partial differential equation through a boundary condition h:

$$(1)\qquad \begin{cases} u_t(t,x) + u_{xxxx}(t,x) = v(t,x), & t \in (0,T),\ x \in (0,\pi), \\ v_t(t,x) - dv_{xx}(t,x) = 0, & t \in (0,T),\ x \in (0,\pi), \\ u(t,0) = u_{xx}(t,0) = 0, & t \in (0,T), \\ u(t,\pi) = u_{xx}(t,\pi) = 0, & t \in (0,T), \\ v(t,0) = h(t),\ v(t,\pi) = 0, & t \in (0,T). \end{cases}$$

Following the methods introduced in [1], we obtain positive and negative results for approximate- and null-controllability, depending on the diophantine approximation properties of the diffusion coefficient d. In particular, we prove that, if $\sqrt{d}$ has finite *irrationality measure* (also called Liouville-Roth constant), then system (1) is null controllable in any time $T > 0$.

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SPECTRAL INEQUALITIES FOR THE SCHRÖDINGER OPERATOR $-\Delta_x + V(x)$ in $\mathbb{R}^d$

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Joint work with **Gilles Lebeau**.

In this talk, we will first review some classical results on the so-called 'spectral inequalities', which yield a sharp quantification of the unique continuation of the spectral family associated with the Laplace-Beltrami operator in a compact manifold. In a second part, we will discuss how to obtain the spectral inequalities associated to the Schrodinger operator $-\Delta_x + V(x)$, in $\mathbb{R}^d$, in any dimension $d \geq 1$, where $V = V(x)$ is a real analytic potential. In particular, we can handle some long- range potentials.

ASYMPTOTIC ANALYSIS OF OPTIMAL CONTROL PROBLEMS IN OSCILLATORY DOMAINS

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Homogenization is a branch of science where we try to understand microscopic structures via a macroscopic medium. Hence, it has applications in various branches of science and engineering. This study is basically developed from material science in the creation of composite materials though the present application is much far and wide. It has applications in *composite media, porous domains, laminar structures, domains with rapidly oscillating boundaries*, to name a few. The PDE problems posed on such complicated domains lead to the analysis of homogenization. It is a process of understanding the microscopic behavior of an in-homogeneous medium via a homogenized medium. Mathematically, it is a kind of asymptotic analysis. There are various methods developed in the last 50 years to understand the mathematical homogenization theory.

In this talk, we discuss the asymptotic analysis of various optimal control problems defined in domains who boundary is rapidly (highly) oscillating. Such complex domains appears in many real life applications like heat radiators, flows in channels with rough boundaries, propagation of electro-magnetic waves in regions having rough interface, absorption diffusion in biological structures, acoustic vibrations in medium with narrow channels etc. In the first part, we briefly present the work which we are carrying out in my group (see and later we present some specific results. We introduce the so called unfolding operators which we have developed for the problems under study through which we characterize the optimal controls. Finally, we do a homogenization process and obtain the limit control problem.

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CONTROL OF BLOWUP AND APPROXIMATE OPTIMALITY CONDITIONS FOR THE WAVE EQUATIONS

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The model problem we study is a semilinear wave equation to which we add a control u in a source function and on the boundary:

$$x_{tt} - \Delta x = f(t, z, x, u) \ \text{ in } \mathbb{R}^+ \times D,$$

$$x(t, z) = u(t, z) \quad \text{on } \mathbb{R}^+ \times \ \partial D,$$

$$x(0, z) = v_0(z), \ x_t(0, z) = v_1(z).$$

This type of problems arise in a great variety of situations. In the paper we propose the problem of controlling a system which may blow up in finite time. We want to minimize the blowup time. To this effect sufficient optimality conditions for controlled blowup time are derived in terms of new dynamic programming methodology. We define ε-optimal value function and we construct sufficient ε-optimal conditions for that function again in terms of new dynamic programming inequality.

MINIMAL CONTROL TIME FOR ONE-DIMENSIONAL FIRST-ORDER HYPERBOLIC SYSTEMS

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Joint work with **Long Hu**.

The goal of this talk is to present some recent results in [2] concerning the exact controllability of one-dimensional first-order linear hyperbolic systems when all the controls are acting on the same side of the boundary. We show that the minimal time needed to control the system is given by an explicit and easy-to-compute formula with respect to all the coupling parameters of the system. The proof relies on the introduction of a canonical UL-decomposition and the compactness-uniqueness method.

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OPTIMAL RESOURCES CONFIGURATIONS IN POPULATION DYNAMICS

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Joint work with **Jimmy Lamboley, Antoine Laurain, and Grégoire Nadin** .

In this work, we are interested in the analysis of optimal resources configurations (typically foodstuff) necessary for a species to survive. For that purpose, we use a logistic equation to model the evolution of population density involving a term standing for the heterogeneous spreading (in space) of resources. The principal issue investigated in this talk writes: How to spread in an optimal way resources in a closed habitat? This problem can be recast as the one of minimizing the principal eigenvalue of an operator with respect to the domain occupied by resources, under a volume constraint. By using symmetrization techniques, as well as necessary optimality conditions, we prove new qualitative results on the solutions. In particular, we investigate the optimality of balls.

EXACT CONTROLLABILITY OF NONLINEAR HEAT EQUATIONS IN SPACES OF ANALYTIC FUNCTIONS

Lionel Rosier
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Joint work with **Camille Laurent**.

It is by now well known that the use of Carleman estimates allows to establish the controllability to trajectories of nonlinear parabolic equations. However, by this approach, it is not clear how to decide whether a given function is indeed reachable. That issue has obtained very recently almost sharp results in the linear case (see [4, 1, 2]). In this talk, we investigate the set of reachable states for a nonlinear heat equation in dimension one. The nonlinear part is assumed to be an analytic function of the spatial variable x, the unknown y, and its derivative y_x. By investigating carefully a nonlinear Cauchy problem in x in some space of Gevrey functions, and the relationship between the jet of space derivatives and the jet of time derivatives, we derive an exact controllability result for small initial and final data that can be extended as analytic functions on some ball of the complex plane. It time allows, works in progress about the reachable states for KdV and for ZK will be outlined.

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OPTIMAL APPROXIMATION OF INTERNAL CONTROLS FOR A WAVE-TYPE PROBLEM WITH FRACTIONAL LAPLACIAN USING FINITE-DIFFERENCE METHOD

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Joint work with **Pierre Lissy**.

We consider a finite-difference semi-discrete scheme for the approximation of internal controls of a one-dimensional evolution problem of hyperbolic type involving the spectral fractional Laplacian. The continuous problem is controllable in arbitrary small time. However, the high frequency numerical spurious oscillations lead to a loss of the uniform (with respect to the mesh size) controllability property of the semi-discrete model in the natural setting. For all initial data in the natural energy space, if we filter the high frequencies of these initial data in an optimal way, we restore the uniform controllability property in arbitrary small time. Even if the initial condition is filtered, the control will excite all frequencies. This creates a lot of technical difficulties, because the spectral is not uniform with respect to the discretization step h. The proof is mainly based on a (non-classic) moment method. For more general uniform controllability results by using filtered spaces and resolvent estimates, the interested reader is referred to [2, 3, 7, 8].

Mathematically speaking, our model can be seen as an intermediate case between the cases of the wave equation and the beam equation. Our strategy consists of an appropriate filtering technique, introduced in [5] and notably used in [1, 4, 6] in the context of wave or beam equation, which consists in relaxing the control requirement by controlling only the low-frequency part of the solution. This approach will be considered here.

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ON BOUNDARY CONTROL OF THE POISSON EQUATION WITH THE THIRD BOUNDARY CONDITION

Amjad Tuffaha
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Joint work with **Alip Mohamed**.

In this talk, we consider an optimal control problem involving the Poisson equation on the unit disk in $\mathbb{C}$ subject to the third boundary condition and where the control is imposed on the boundary. We use complex analytic methods to prove existence and uniqueness of the control when the parameter λ is a nonzero complex number but not a negative integer (not an eigenvalue). Otherwise, due to multiplicity of solutions to the underlying problem, when λ is a negative integer, controllability could only be obtained if proper additional conditions on the boundary are imposed.

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DIFFERENTIAL ALGEBRAIC PORT-HAMILTONIAN EQUATIONS

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Joint work with **Volker Mehrmann**.

Port-Hamiltonian (pH) models can be used to describe physical systems which interact with their environment. Examples include ordinary and partial differential equations. By now the theory is quite complete with results ranging from control, approximation, and well-posedness of partial differential equations, [2]. Recently, the theory has been extended to differential equations with constraints, i.e., differential algebraic equations (DAE). We refer to [3] for the algebraic set-up of these systems and to [1] for (time-varying) DAE's with a finite-dimensional state space. Following on this, we study (time-invariant) port-Hamiltonian DAE on an infinite-dimensional state space. Our main focus is to show existence of (mild) solutions for this class of systems. So we consider the following abstract differential equation

$$E\dot{x}(t) = AQx(t), \qquad x(0) = x_0$$

where E and Q are bounded operators on X satisfying $E^*Q = Q^*E$, and A is the infinitesimal generator of a contraction semigroup on the Hilbert space X. Since A is the the infinitesimal generator of a contraction semigroup, the above equation possesses a solution when $E = Q = I$. If E and Q are are non-invertible, then this needs not to hold. We present some sufficient conditions under which the above DAE possesses a unique solution. Furthermore, we show that our results are related to boundary triplets and passive systems.

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