DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D23

AGH University of Science and Technology

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D2

DIFFERENTIABILITY OF THE FLOW FOR BV VECTOR FIELDS

Stefano Bianchini
SISSA, Italy

We show that the Regular Lagrangian Flow $X(t, y)$ generated by nearly incompressible BV vector fields admits a derivative $\nabla X(t, y)$ in the sense of measure. This matrix satisfies the ODE

$$\frac{d}{dt}\nabla X(t, y) = (D\mathbf{b}(t))_y \nabla X(t-, y)$$

where $(D\mathbf{b})_y$ is the disintegration of the measure $\int D\mathbf{b}(t)dt$ w.r.t. the trajectories $X(t, y)$.

ON THE LARGE TIME BEHAVIOR OF SOLUTIONS TO BIRTH AND SPREAD TYPE EQUATIONS

Yoshikazu Giga
University of Tokyo, Japan
Joint work with **Hiroyoshi Mitake, Takeshi Ohtsuka, and Hung V. Tran**.

We consider a level-set eikonal-curvature flow equation with an external force. Such a problem is considered as a model to describe an evolution of height of crystal surface by two-dimensional nucleation or possibly some class of growths by screw dislocations. For applications, it is important to estimate growth rate. Without an external source term the solution only spreads horizontally and does not grow vertically so the source term plays a key role for the growth.

Although the large time behavior of parabolic equations are well studied, the equations we study are degenerate parabolic equations where no diffusion effect exists in the normal to each level-set of a solution. Thus, very little is known even for growth rate. Our goal is to describe our recent progress on such type of problems. Ealier results are presented in the paper by H. Mitake, H.V. Tran and the lecturer published in SIAM Math. Anal. in 2016. A review paper is published in Proc. Int. Cong. of Math. in 2018.

In this talk, we first show the existence of asymptotic speed called growth rate. We also discuss asymptotic profile as well as estimates on growth rate.

THE TWO HYPERPLANE CONJECTURE

David Jerison
Massachusetts Institute of Technology, USA

I will introduce a conjecture that I call the *Two Hyperplane Conjecture*, saying that an isoperimetric surface that divides a convex body in half by volume is trapped between parallel hyperplanes. Emanuel Milman has shown that in its strongest, dimension-independent form, my conjecture implies the *Hyperplane Conjecture* of Kannan, Lovász and Simonovits in theoretical computer science, which says that the area of such an isoperimetric surface is comparable, by an absolute constant independent the convex body and its dimension, to the area of some hyperplane dividing the convex body in half. Their conjecture is closely related to several famous unsolved problems in high dimensional convex geometry. But unlike the hyperplane conjecture, the two-hyperplane conjecture has significance even in low dimensions.

I will relate the conjecture to qualitative and quantitative connectivity properties and regularity of area-minimizing surfaces, free boundaries and level sets of eigenfunctions, and report on work in progress with Guy David. The main theme of the talk is that the level sets of least energy solutions to scalar variational problems should be as simple as possible, but no simpler.

ON THE NONLINEAR STABILITY OF BLACK HOLES

Sergiu Klainerman
Princeton University, USA

Black holes are precise mathematical solutions of the Einstein field equations mainly represented by the famous two parameter Kerr family including, as a particular case, the Schwarzschild solution. To correspond to physical reality, i.e. to be more than mathematical artifacts, these solutions have to be stable under small perturbations. While there is today no doubt concerning the physical reality of black holes, based both on observational data and numerical simulations, an actual proof of stability remains a fundamental challenge of Mathematical and Geometric Analysis.

In my talk I will formulate the precise mathematical problem of the nonlinear stability of the Kerr family and describe the main results known so far. In the second part of the talk I will describe my recent result with J. Szeftel "Global Nonlinear Stability of Schwarzschild Spacetime under Polarized perturbations" - arXiv:1711.07597. The result establishes the full nonlinear stability of Schwarzschild spacetime under axially symmetric, polarized perturbations, i.e. stability of solutions of the Einstein vacuum equations for asymptotically flat $1 + 3$ dimensional Lorentzian metrics which admit a hyper-surface orthogonal space-like Killing vector-field with closed orbits. While building on the remarkable advances made in last 15 years on establishing quantitative linear stability, the paper introduces a series of new ideas among which we emphasize the *general covariant modulation* (GCM) procedure which allows us to construct, dynamically, the center of mass frame of the final state. The mass of the final state itself is tracked using the well known Hawking mass relative to a well adapted foliation itself connected to the center of mass frame. Our work here is the first to prove the nonlinear stability of Schwarzschild in a restricted class of nontrivial perturbations. To a large extent, the restriction to this class of perturbations is only needed to ensure that the final state of evolution is another Schwarzschild space.

ZERO SETS OF LAPLACE EIGENFUCNTIONS

Aleksandr Logunov
Princeton University, USA

We will discuss geometrical and analytic properties of zero sets of harmonic functions and eigenfunctions of the Laplace operator. For harmonic functions on the plane there is an interesting relation between local length of the zero set and the growth of harmonic functions. The larger the zero set is, the faster the growth of harmonic function should be and vice versa. Laplace eigenfunctions on two dimensional sphere are restrictions of homogeneous harmonic polynomials of three variables onto the sphere. Zero sets of such functions are unions of smooth curves with equiangular intersections. Topology of the zero set could be quite complicated, but the total length of the zero set of any spherical harmonic of degree n is comparable to n.

References

[1] A. Logunov, E. Malinnikova, *Review of Yau's conjecture on zero sets of Laplace eigenfunctions*, to appear in Current Developments in Mathematics.

[2] A. Logunov, E. Malinnikova, *Quantitative propagation of smallness for solutions of elliptic equations*, Proceedings of the International Congress of Mathematicians–Rio de Janeiro, 2357–2378, 2018.

[3] A. Logunov, E. Malinnikova, *Nodal sets of Laplace eigenfunctions: estimates of the Hausdorff measure in dimension two and three*, 50 Years with Hardy Spaces, A Tribute to Victor Havin, 333–344, 2018.

[4] A. Logunov, E. Malinnikova, *Lecture notes on quantitative unique continuation for solutions of second order elliptic equations*, to appear in IAS/Park City Mathematics series, AMS.

[5] A. Logunov, 2018, *Nodal sets of Laplace eigenfunctions: polynomial upper estimates of the Hausdorff measure*, Annals of Mathematics, **187(1)**, 221–239.

[6] A. Logunov, 2018, *Nodal sets of Laplace eigenfunctions: proof of Nadirashvili's conjecture and of the lower bound in Yau's conjecture*, Annals of Mathematics, **187(1)**, 241–262.

EFFECTIVE BEHAVIOR OF RANDOM MEDIA

Felix Otto
Max Planck Institute for Mathematics in the Sciences, Germany

In engineering applications, heterogeneous media are often described in statistical terms. This partial knowledge is sufficient to determine the effective, i.e. large-scale behavior. This effective behavior may be inferred from the Representative Volume Element (RVE) method. I report on last years' progress on the quantitative understanding of what is called *stochastic homogenization of elliptic partial differential equations*: optimal error estimates of the RVE method, leading-order characterization of fluctuations, effective multipole expansions. Methods connect to elliptic regularity theory and to concentration of measure arguments.

IMPLICITLY CONSTITUTED FLUID FLOW MODELS: ANALYSIS AND APPROXIMATION

Endre Süli
University of Oxford, UK

Classical models describing the motion of Newtonian fluids, such as water, rely on the assumption that the Cauchy stress is a linear function of the symmetric part of the velocity gradient of the fluid. This assumption leads to the Navier-Stokes equations. It is known however that the framework of classical continuum mechanics, built upon a simple linear constitutive equation for the Cauchy stress, is too narrow to describe inelastic behavior of solid-like materials or viscoelastic properties of materials. Our starting point in this work is therefore a generalization of the classical framework of continuum mechanics, called the implicit constitutive theory, which was proposed recently in a series of papers by Rajagopal. The underlying principle of the implicit constitutive theory in the context of viscous flows is the following: instead of demanding that the Cauchy stress is an explicit (and, in particular, linear) function of the symmetric part of the velocity gradient, one may allow a nonlinear, implicit and not necessarily continuous relationship between these quantities. The resulting general theory therefore admits non-Newtonian fluid flow models with implicit and possibly discontinuous power-law-like rheology.

We develop the analysis of finite element approximations of implicit power-law-like models for viscous incompressible fluids. The Cauchy stress and the symmetric part of the velocity gradient in the class of models under consideration are related by a, possibly multi-valued graph. Using a variety of weak compactness techniques, we show that when the graph of the stress-strain relationship is maximal monotone a subsequence of the sequence of finite element solutions converges to a weak solution of the problem as the discretization parameter, measuring the granularity of the finite element triangulation, tends to zero. When the graph is nonmonotone, a subsequence of the sequence of finite element solutions is shown to converge to a gradient Young-measure solution of the problem. A key new technical tool in the analysis is a finite element counterpart of the Acerbi-Fusco Lipschitz truncation of Sobolev functions. The talk is based on a series of papers with Miroslav Bulíček and Josef Málek (Prague), Miles Caddick (Oxford), Lars Diening (Bielefeld), Christian Kreuzer (Dortmund), and ongoing research with Alexei Gazca-Orozco (Oxford) and Tabea Tscherpel (Aachen).

References

[1] M. Bulíček, J. Málek, E. Süli, *Existence of global weak solutions to implicitly constituted kinetic models of incompressible dilute homogeneous polymers*, Commun. Part. Diff. Eq. **38(5)** (2013), 882-924.

[2] M. Caddick, E. Süli, *Numerical approximation of Young-measure solutions to parabolic systems of forward-backward type*, Appl. Anal. Discr. Math., Accepted for publication on 17/02/2019.

[3] L. Diening, C. Kreuzer, E. Süli, *Finite element approximation of steady flows of incompressible fluids with implicit power-law-like rheology*, SIAM J. Numer. Anal. **51(2)** (2013), 984-1015.

[4] C. Kreuzer, E. Süli, *Adaptive finite element approximation of steady flows of incompressible fluids with implicit power-law-like rheology*, ESAIM: Math. Model. Numer. Anal. **50(5)** (2016), 1333-1369.

[5] E. Süli, T. Tscherpel, *Fully discrete finite element approximation of unsteady flows of implicitly constituted incompressible fluids*, IMA J. of Numer. Anal., https://doi.org/10.1093/imanum/dry097.

GLOBAL ANALYSIS VIA MICROLOCAL TOOLS: FREDHOLM THEORY IN NON-ELLIPTIC SETTINGS

András Vasy
Stanford University, USA
Joint work with **Peter Hintz**.

In this lecture I will describe a microlocal framework for the Fredholm analysis of non-elliptic problems both on manifolds without boundary and manifolds with boundary, introduced in [8] and extended in [6] and various other works. Examples in which such a framework (or a similar framework) has recently been useful include wave propagation on black hole spacetimes, which is the key analytic ingredient for showing the stability of black holes [5, 4], analysis of the resolvent of the generator of the flow for dynamical systems [2], which is the key tool for the analysis of the Ruelle zeta function [1], Feynman propagators in quantum field theory [3, 9] and inverse problems, namely boundary rigidity and tensor tomography [7].

References

[1] Semyon Dyatlov and Maciej Zworski, *Dynamical zeta functions for Anosov flows via microlocal analysis*, Ann. Sci. Éc. Norm. Supér. (4) **49(3)** (2016), 543–577.

[2] Frédéric Faure and Johannes Sjöstrand, *Upper bound on the density of Ruelle resonances for Anosov flows*, Comm. Math. Phys., **308(2)** (2011), 325–364.

[3] Jesse Gell-Redman, Nick Haber and András Vasy, *The Feynman propagator on perturbations of Minkowski space*, Comm. Math. Phys. **342(1)** (2016), 333–384.

[4] Peter Hintz and András Vasy, *Linear stability of slowly rotating Kerr black holes*, Preprint, arXiv:1906.00860, 2019.

[5] Peter Hintz and András Vasy, *The global non-linear stability of the Kerr–de Sitter family of black holes*, Acta mathematica, **220** (2018), 1–206.

[6] Peter Hintz and András Vasy, *Semilinear wave equations on asymptotically de Sitter, Kerr–de Sitter and Minkowski spacetimes*, Anal. PDE **8(8)** (2015), 1807–1890.

[7] Plamen Stefanov, Gunther Uhlmann and András Vasy, *Local and global boundary rigidity and the geodesic X-ray transform in the normal gauge* Preprint, arXiv:1702.03638, 2017.

[8] András Vasy, *Microlocal analysis of asymptotically hyperbolic and Kerr–de Sitter spaces (with an appendix by Semyon Dyatlov)*, Invent. Math., 194(2):381–513, 2013.

[9] András Vasy and Michał Wrochna, *Quantum Fields from Global Propagators on Asymptotically Minkowski and Extended de Sitter Spacetimes*, Ann. Henri Poincaré **19(5)** (2018), 1529–1586.

THE VORTEX FILAMENT EQUATION, THE TALBOT EFFECT, AND NON-CIRCULAR JETS

Luis Vega
University of the Basque Country & BCAM, Spain
Joint work with **Valeria Banica and Francisco De La Hoz**.

We will propose the vortex filament equation as a possible toy model for turbulence, in particular because of its striking similarity to the dynamics of non-circular jets. This similarity implies the existence of some type of Talbot effect due to the interaction of non-linear waves that propagate along the filament. Another consequence of this interaction is the existence of a new class of multi-fractal sets that can be seen as a generalization of the graph of Riemann's non-differentiable function. Theoretical and numerical arguments about the transfer of energy will be also given.

POPULATION DYNAMICS AND CONTROL

Enrique Zuazua
DeustoTech, Bilbao, Basque Country & Universidad Autónoma de Madrid, Spain

Population dynamics is an old subject. Classical models in this field are written in terms of reaction-diffusion equations.

There is a wide literature concerning the dynamical properties of those systems. But much less is known from a control perspective. And control constitutes the ultimate proof of our understanding of a process.

This lecture will be devoted to present two recent results in this area. We first consider a bistable reaction-diffusion arising in the modelling of bilingual populations and then address models involving age structuring and spatial diffusion (of Lotka-McKendrick type).

As we shall see, both aspects require of an in depth understanding of the dynamics of the systems under consideration.

We shall present sharp results on our ability to steer the dynamics of those systems to a prescribed final configuration. Some open problems and future directions of research will also be presented.

This lecture is based on recent joint work in collaboration with D. Maity, C. Pouchol, E. Trélat, M. Tucsnak and J. Zhu.

References

[1] E. Trélat, J. Zhu, E. Zuazua, *Allee optimal control of a system in ecology*, Mathematical Models and Methods in Applied Sciences **28(9)** (2018), 1665-1697.

[2] C. Pouchol, E. Trélat, E. Zuazua, *Phase portrait control for 1D monostable and bistable reaction-diffusion equations*, Nonlinearity, to appear (hal-01800382).

[3] D. Maity, M. Tucsnak, E. Zuazua, *Sharp Time Null Controllability of a Population Dynamics Model with Age Structuring and Diffusion*, J. Math pures et appl., to appear (hal-01764865).

TALKS OF SESSION D23

EXACT SPLITTINGS OF LINEAR QUADRATIC PDES

Joackim Bernier
Université Paul Sabatier, France
Joint work with **Paul Alphonse, Fernando Casas, Nicolas Crouseilles, and Yingzhe Li**.

Usually, the higher the order of a splitting method is, the larger its number of steps is. However, we will see that for many linear ODEs, the resolution of an inverse problem provides an exact splitting, involving some modified vector fields, with the same number of steps as the usual low order methods (i.e. Lie or Strang splittings). Applying the Fourier integral operators theory (developed by Hörmander in [4]), we will see how these decompositions can be transposed at the level of the quadratic linear PDEs. I will show how this construction provides some new efficient splittings for many PDEs like the Schrödinger equations in rotating frames or the Vlasov equations with a rotation motion.

References

[1] P. Alphonse, J. Bernier, *Polar decomposition of semigroups generated by non-selfadjoint quadratic differential operators and regularizing effects*, in preparation.

[2] J. Bernier, F. Casas, N. Crouseilles, *Splitting methods for rotations: application to Vlasov equations*, preprint available on HAL.

[3] J. Bernier, Y. Li, N. Crouseilles, *Splitting methods for Schrödinger equations in rotating frames*, in preparation.

[4] L. Hörmander, *Symplectic classification of quadratic forms, and general Mehler formulas*, Math. Z., **219** (1995), 413-449.

A FICTITIOUS DOMAIN APPROACH TO FLUID-STRUCTURE INTERACTION PROBLEMS

Daniele Boffi
University of Pavia, Italy

We review a distributed Lagrange multiplier formulation of the Finite Element Immersed Boundary Method for the numerical approximation of the interaction between fluids and solids (see [1, 2]). The discretization of the problem leads to a mixed problem for which a rigorous stability analysis is provided. Optimal convergence estimates are proved for its finite element space discretization. The model, originally introduced for the coupling of incompressible fluids and solids, can be extended to include the simulation of compressible structures [3].

Recent research [4] investigates several time marching strategies for the proposed method.

References

[1] D. Boffi, N. Cavallini, L. Gastaldi, The Finite Element Immersed Boundary Method with Distributed Lagrange multiplier, SIAM Journal on Numerical Analysis **53(6)** (2015), 2584–2604.

[2] D. Boffi, L. Gastaldi, *A fictitious domain approach with distributed Lagrange multiplier for fluid-structure interactions*, Numerische Mathematik **135(3)** (2017), 711–732.

[3] D. Boffi, L. Gastaldi, L. Heltai, *A distributed Lagrange formulation of the Finite Element Immersed Boundary Method for fluids interacting with compressible solids*, in Mathematical and Numerical Modeling of the Cardiovascular System and Applications, D. Boffi, L. Pavarino, G. Rozza, S. Sacchi, C. Vergara eds., SEMA SIMAI Springer Series **16** (2018), 1–21.

[4] D. Boffi, L Gastaldi, S. Wolf, *Higher order time stepping schemes for fluid-structure interaction problems*, in preparation.

A UNIFIED PDE/NUMERICAL FRAMEWORK FOR NONLOCAL AND LOCAL EQUATIONS OF POROUS MEDIUM TYPE

Félix del Teso
BCAM, Spain

We consider the following problem of porous medium type:

$$\begin{cases} \partial_t u(x,t) - (L^\sigma + \mathcal{L}^\mu)[\varphi(u)](x,t) = f(x,t), & (x,t) \in \mathbb{R}^N \times (0,\infty), \\ u(x,t) = u_0(x), & x \in \mathbb{R}^N, \end{cases}$$

where $\varphi : \mathbb{R}^N \to \mathbb{R}$ is continuous and nondecreasing, and

$$L^\sigma[v](x) = \mathrm{Tr}\left(\sigma\sigma^T D^2 v(x)\right), \qquad \textbf{(local diffusion)}$$
$$\mathcal{L}^\mu[v](x) = \mathrm{P.V.}\int_{|z|>0} (v(x+z) - v(x))\, d\mu(z), \quad \textbf{(nonlocal diffusion)}$$

with $\sigma \in \mathbb{R}^{N\times p}$ and μ symmetric measure s.t. $\int \min\{|z|^2, 1\} d\mu(z) < +\infty$.

We will present a general overview of some of the results obtained in collaboration with J. Endal and E.R. Jakobsen:

• Uniqueness of distributional solutions.

• Continuous dependence on $L^\sigma + \mathcal{L}^\mu$, φ and u_0.

• Unified theory of monotone numerical schemes of finite difference type. Here we use the fact that operators in the class of $\mathcal{L}^\mu$ includes discretizations of $L^\sigma + \mathcal{L}^\mu$. This fact allows us to use a pure PDE approach.

• We also propose a branch of discretizations and schemes and analyze their accuracy.

• As a consequence of numerics, we obtain existence of distributional solutions together with interesting properties like L^1-contraction, $C([0,T], L^1_{\mathrm{loc}}(\mathbb{R}^N))$ regularity, energy estimates, ...

A STATISTICAL LEARNING APPROACH FOR PARAMETRIC PDES

Martin Eigel
WIAS, Germany
Joint work with **Reinhold Schneider and Philipp Trunschke**.

Parametric PDEs (as encountered in the popular field of Uncertainty Quantification) are computationally complex due to the high dimensionality of the models describing random data. Common numerical approaches are Monte Carlo methods for statistical quantities of interest and functional approximations, representing the entire solution manifold in some function space. Assuming sufficient regularity (or sparsity), the latter attain high theoretical convergence rates. In practice, this can be realised e.g. by employing some kind of (a posteriori) error control in the computations. However, the implementation usually is non-trivial and does not generalise easily.

We examine a non-intrusive "Variational Monte Carlo" (VMC) method based on statistical learning theory. This provides a combination of deterministic and statistical convergence results. The Galerkin solution can be computed with high probability using a tensor recovery algorithm on a training set of generated solution realisations. The representation in efficient hierarchical tensor formats tames the "curse of dimensionality". Similarly, a residual a posteriori error estimator can be reconstructed easily, steering all discretisation parameters.

References

[1] M. Eigel, R. Schneider, P. Trunschke, S. Wolf, *Variational Monte Carlo - Bridging Concepts of Machine Learning and High Dimensional PDEs*, Advances in Comp. Math., (to be published 2019).

ROBUST SOLVERS FOR HIGH-CONTRAST MULTISCALE PROBLEMS

Juan Galvis
National University of Colombia, Colombia

We present some recent developments in the numerical approximation of elliptic partial differential equations with high-contrast multiscale coefficients. In particular we review recently introduced robust upscaling technique known as the generalized multiscale finite element method (GMsFEM). We also present the design of robust two-levels domain decomposition methods that use the GMsFEM method as a second level. In order to show the benefits of using the proposed methodology several appliations are cosidered: two-phase flow in high-contrast multiscale porous media, the free boundary dam problem in heterogeneous media and an elasticity problem in topology optimization.

References

[1] J. Galvis, C. Vásquez, L.F. Contreras, *Numerical upscaling of the free boundary dam problem in multiscale high-contrast media*, Submitted. 2019.

[2] E. Abreu, C. Diaz, J. Galvis, *A convergence analysis of Generalized Multiscale Finite Element Methods*, 1990.

[3] B. Lazarov, S. Serrano, M. Zambrano, J. Galvis, *Fast multiscale contrast independent preconditioner for linear elastic topology optimization problems*, in preparation, 2019.

[4] Y. Efendiev, J. Galvis, T.Y. Hou, *Generalized multiscale finite element methods (GMsFEM)*, Journal of Computational Physics **251** (2013), 116-135.

APPROXIMATION OF DIRICHLET BOUNDARY CONTROL PROBLEM USING ENERGY SPACES

Thirupathi Gudi
Indian Institute of Science, India
Joint work with **Sudipto Chowdhury and Akambadath K. Nandakumaran**.

In this talk, we present an alternative energy space based approach for formulating the Dirichlet boundary control problem and then propose a finite element based numerical method for approximating its solution numerically. A priori error estimates of optimal order in the energy norm as well as in the L_2 norm will be discussed. Furthermore, we discuss on deriving a reliable and efficient a posteriori error estimator using an auxiliary problem for adaptive mesh refinement. The theoretical results will be illustrated by some numerical experiments.

References

[1] S. Chowdhury, T. Gudi, A.K. Nandakumaran, *Error bounds for a Dirichlet boundary control problem based on energy spaces*, Journal Math. Comp. **86** (2017), 1103-1126.

SMOOTH FINITE ELEMENT EXACT SEQUENCE ON POWELL-SABIN SPLITS

Johnny Guzmán
Brown University, USA
Joint work with **Anna Lischke and Michael Neilan**.

Starting with C^1 finite elements on a Powell-Sabin split we show how to construct a de Rham exact sequence. The lowest order FEM in our family of FEM is piecewise quadratics. An interesting feature in the exact sequence is that Powell-Sabin splits introduce singular vertices and naturally we have to constrain the space of 2-forms in two dimensions. We will indicate possible generalization to higher dimensions.

ANALYSIS OF TENSOR APPROXIMATION SCHEMES FOR CONTINUOUS FUNCTIONS

Helmut Harbrecht
University of Basel, Switzerland
Joint work with **Michael Griebel**.

This talk is concerned with the analysis of tensor approximation schemes for continuous functions in high dimensions. We assume that the function to be approximated lies in an isotropic Sobolev space and discuss the cost when approximating this function in the continuous analogue of the Tucker tensor format or of the tensor train format. We especially show that the cost of both approximations are dimension-robust when the Sobolev space under consideration provides appropriate weights.

References

[1] M. Griebel, H. Harbrecht, *Analysis of tensor approximation schemes for continuous functions*, arXiv:1509.09058, (2019).

THE NUMERICAL SOLUTION OF NONLINEAR SCHRÖDINGER EQUATIONS WITH APPLICATIONS TO SUPERFLUIDITY

Patrick Henning
Royal Institute of Technology (KTH), Sweden
Joint work with **Daniel Peterseim and Johan Wärnegård**.

In this talk we consider the numerical solution of a class of nonlinear Schrödinger equations by Galerkin finite elements in space and a mass- and energy-conserving variant of the Crank-Nicolson method in time. The usage of finite elements becomes necessary if the equation contains terms that dramatically reduce the overall regularity of the exact solution. Examples of such terms are rough potentials or disorder potentials as appearing in many physical applications. We present some analytical results that show how the reduced regularity of the exact solution could affect the expected convergence rates and how it results in possible coupling conditions between the spatial mesh size and the time step size. We will also demonstrate the importance of numerical energy-conservation in applications with low-regularity by simulating the phase transition of a Mott insulator into a superfluid.

References

[1] P. Henning, D. Peterseim, *Crank-Nicolson Galerkin approximations to nonlinear Schrödinger equations with rough potentials*, M3AS Math. Models Methods Appl. Sci. **27(11)** (2017), 2147-2184.

[2] P. Henning, J. Wärnegård, *Numerical comparison of mass-conservative schemes for the Gross-Pitaevskii equation*, ArXiv e-print 1804.10547, 2018.

[3] P. Henning, D. Peterseim, *Sobolev gradient flow for the Gross-Pitaevskii eigenvalue problem: global convergence and computational efficiency*, ArXiv e-print 1812.00835, 2018.

ON TRAFFIC MODELING AND THE BRAESS PARADOX

Helge Holden
Norwegian University of Science and Technology, Norway
Joint work with **Nils Henrik Risebro and Rinaldo Colombo**.

We will discuss models for vehicular traffic flow on networks. The models include both the Lighthill-Whitham-Richards (LWR) model and Follow-the-Leader (FtL) models. Emphasis will be on the Braess paradox [1] in which adding a road to a traffic network can make travel times worse for all drivers, and we will show one way of studying the Braess paradox with an LWR model [2].

Furthermore, we will show how we can consider the FtL model as a discretization of the LWR model [3, 4]. Finally, we will also discuss a novel model for multi-lane traffic [5].

References

[1] D. Braess, *Über ein Paradoxon aus der Verkehrsplanung*, Unternehmensforschung **12** (1968), 258-268.

[2] R. Colombo, H. Holden, *On the Braess paradox with nonlinear dynamics and control theory*, J. Optimization Theory and Appl. **168** (2016), 216-230.

[3] H. Holden, N.H. Risebro, *Continuum limit of Follow-the-Leader models - a short proof*, Discrete Contin. Dyn. Syst. **38** (2018), 715-722.

[4] H. Holden, N.H. Risebro, *Follow-the-Leader models can be viewed as a numerical approximation to the Lighthill-Whitham-Richards model for traffic flow*, Networks & Heterogeneous Media **13** (2018), 409-421.

[5] H. Holden, N.H. Risebro, *Models for dense multilane vehicular traffic*, arXiv:1812.01361.

SPECTRAL RELATIONS OF THE GENERATOR TO ALPHA-STABLE PROCESSES AND RELATED SPECTRAL METHODS

Yanghong Huang
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Joint work with **Xiao Wang**.

The majority of special functions and orthogonal polynomials are well-known to be associated with second order differential equations arising from mathematical physics, and are usually have to be extended to Merjie G function or Fox H function. In this talk, classical Jacobi polynomials are shown to establish spectral relations of the generator of alpha-stable processes, generalising the fractional Laplacian in one dimension to the non-symmetric case. The resulting spectral relations will be used to characterised the singularity near the boundary and the regularity of the solution to the Dirichlet problem, together with the development of a higher order spectral methods.

TIME-STEPPING SCHEMES FOR FRACTIONAL DIFFUSION

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Overall the last decade, a large number of time stepping schemes have been developed for time-fractional diffusion problems. These schemes can be generally divided into: finite difference type, convolution quadrature type and discontinuous Galerkin methods. Many of these methods are developed by assuming that the solution is sufficiently smooth, which however is generally not true. In this talk, I will describe recent works in analyzing and developing robust numerical schemes that do not assume solution regularity directly, but only data regularity.

A POSTERIORI ERROR CONTROL AND ADAPTIVITY FOR SCHRÖDINGER EQUATIONS

Theodoros Katsaounis
KAUST, Saudi Arabia & University of Crete, Greece
Joint work with **Irene Kyza**.

In this talk I will present some recent results on aposteriori error estimation for linear and nonlinear Schrodinger equations. We use finite element discretizations and the Crank Nicolson time stepping scheme. For the derivation of the estimates we use the reconstruction technique and linear and nonlinear stability arguments as in the continuous problem. Based on these aposteriori estimators we further design and analyse a time-space adaptive algorithm. Various numerical experiments verify and complement our theoretical results.

ROBUST PRECONDITIONING FOR DISCONTINUOUS GALERKIN DICRETIZATIONS OF DIFUSSION PROBLEM WITH HIGH CONTRAST COEFFICIENTS

Piotr Krzyżanowski
University of Warsaw, Poland

We consider a diffusion problem in a heterogeneous medium, with prescribed transmission properties. We discuss preconditioners for iterative solutions of algebraic systems arising from problem discretizations of discontinuous Galerkin type.

In particular, a diffusion problem through a thin membrane is discussed. A nonoverlapping domain decomposition based preconditioner is introduced, and its convergence properties are discussed and verified in numerical experiments. In particular, the convergence rate is shown independent of the contrast in the diffusion coefficient, the number of inclusions and of the transmission parameter as well.

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS ON MANIFOLDS AND POINT CLOUDS

Shingyu Leung
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Joint work with **Hongkai Zhao, Meng Wang, and Ningchen Ying**.

We present recent numerical methods for solving partial differential equations on manifolds and point clouds. In the first part of the talk, we introduce a new and simple discretization, named the Modified Virtual Grid Difference (MVGD), for numerical approximation of the Laplace-Beltrami operator on manifolds sampled by point clouds. We first introduce a local virtual grid with a scale adapted to the sampling density centered at each point. Then we propose a modified finite difference scheme on the virtual grid to discretize the LB operator. The new discretization provides more diagonal dominance to the resulting linear system and improves its conditioning. In the second part, we present a local regularized least squares radial basis function (RLS-RBF) method for solving partial differential equations on irregular domains or on manifolds. The idea extends the standard RBF method by replacing the interpolation in the reconstruction with the least squares fitting approximation.

References

[1] M. Wang, S. Leung and H. Zhao, *Modified Virtual Grid Difference for Discretizing the Laplace-Beltrami Operator on Point Clouds*, SIAM J. Sci. Comput. **40** (2018).

[2] N. Ying and S. Leung, *A Local Regularized Least Squares Radial Basis Function (RLS-RBF) Method for Differential Operators in Meshless Domain*, in preparation.

MULTILEVEL QUADRATURE FOR ELLIPTIC PARAMETRIC PARTIAL DIFFERENTIAL EQUATIONS

Michael Multerer
USI Lugano, Switzerland
Joint work with **Michael Griebel and Helmut Harbrecht**.

Multilevel quadrature methods for parametric operator equations such as the multilevel (quasi-) Monte Carlo method are closely related to the sparse tensor product approximation between the spatial variable and the parameter. In this presentation, we employ this fact and reverse the multilevel quadrature method via the sparse grid construction by applying differences of quadrature rules to finite element discretizations of increasing resolution. Besides being algorithmically more efficient if the underlying quadrature rules are nested, this way of performing the sparse tensor product approximation enables the easy use of non-nested and even adaptively refined finite element meshes. Especially, we present an error and regularity analysis of the fully discrete solution, taking into account the effect of polygonal approximations to a curved physical domain and the numerical approximation of the bilinear form. Numerical results in three spatial dimensions are provided to illustrate the approach.

References

[1] M. Griebel, H. Harbrecht and M. Multerer, *Multilevel quadrature for elliptic parametric partial differential equations in case of polygonal approximations of curved domains*, arXiv:1509.09058, 2018.

THE GENERALIZED FRACTIONAL BENJAMIN-BONA-MAHONY EQUATION: ANALYTICAL AND NUMERICAL RESULTS

Gulcin M. Muslu
Istanbul Technical University, Turkey
Joint work with **Goksu Oruc and Handan Borluk**.

The generalized fractional Benjamin-Bona-Mahony (gfBBM) equation models the propagation of small amplitude long unidirectional waves in a nonlocally and nonlinearly elastic medium. The equation involves two fractional terms unlike the well-known fBBM equation. In this talk, we present the local existence and uniqueness of the solutions for the Cauchy problem. The sufficient conditions for the existence of solitary wave solutions are discussed. The Petviashvili method is proposed for the generation of the solitary wave solutions and their evolution in time is investigated by Fourier spectral method, numerically. The efficiency of the numerical methods is tested and the relation between nonlinearity and fractional dispersion will be presented by various numerical experiments.

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EXACT SEQUENCES OF PIECEWISE POLYNOMIALS ON ALFELD SPLITS

Michael Neilan
University of Pittsburgh, USA
Joint work with **Guosheng Fu and Johnny Guzmán**.

We develop exact polynomial sequences on Alfeld splits in any spatial dimension and any polynomial degree. An Alfeld split of a simplex is obtained by connecting the vertices of an n-simplex with its barycenter. We show that, on these triangulations, the kernel of the exterior derivative has enhanced smoothness. Byproducts of this theory include characterizations of

discrete divergence-free subspaces for the Stokes problem, commutative projections, and simple formulas for the dimensions of smooth polynomial spaces.

LOW-REGULARITY INTEGRATORS FOR NONLINEAR SCHRÖDINGER EQUATIONS

Alexander Ostermann
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Joint work with **Frédéric Rousset, Marvin Knöller and Katharina Schratz** .

Nonlinear Schrödinger equations are usually solved by pseudo-spectral methods, where the time integration is performed by splitting schemes or exponential integrators. Notwithstanding the benefits of this approach, its successful application requires additional regularity of the solution. For instance, second-order Strang splitting requires four additional derivatives for the solution of the cubic nonlinear Schrödinger equation. Similar statements can be made about other dispersive equations like the Korteweg–de Vries or the Boussinesq equation.

In this talk, we introduce as an alternative low-regularity Fourier integrators. They are obtained from Duhamel's formula in the following way: first, a Lawson-type transformation eliminates the leading linear term and second, the dominant nonlinear terms are integrated exactly in Fourier space. For cubic nonlinear Schrödinger equations, first-order convergence of such methods only requires the boundedness of one additional derivative of the solution, and second-order convergence the boundedness of two derivatives. For details, see [1, 2].

Moreover, a filtered low-regularity Fourier integrator for the cubic nonlinear Schrödinger equation is presented. This scheme has better convergence rates at low regularity than any known scheme in the literature so far. To prove this superior error behavior, we combine the better local error properties of the new scheme with a stability analysis based on general discrete Strichartz-type estimates. The latter allow us to handle a much rougher class of solutions as the error analysis can be carried out directly at the level of L^2. We are able to establish a global error estimate in L^2 for H^1 solutions, which is roughly of order $\tau^{\frac{1}{2}+\frac{5-d}{12}}$ in dimension $d \leq 3$ with τ denoting the time step size. For details, see [3].

References

[1] M. Knöller, A. Ostermann, K. Schratz, *A Fourier integrator for the cubic nonlinear Schrödinger equation with rough initial data*, Preprint, arXiv:1807.01254, to appear in SIAM J. Numer. Anal. (2019).

[2] A. Ostermann, K. Schratz, *Low regularity exponential-type integrators for semilinear Schrödinger equations*, Found. Comput. Math. **18** (2018), 731-755.

[3] A. Ostermann, F. Rousset, K. Schratz, *Error estimates of a Fourier integrator for the cubic Schrödinger equation at low regularity*, Preprint, arXiv:1902.06779 (2019).

EFFECTIVE COMPRESSIONAL WAVE VELOCITY ESTIMATION FOR POROUS ROCKS

David Pardo
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Joint work with **Ángel Javier Omella, Julen Alvarez-Aramberri, Magdalena Strugaru, Vincent Darrigrand, Carlos Santos, and Héctor González**.

In geophysics, it is of paramount importance to characterize the effective compressional wave velocity of the Earth's crust layers. In this work, we propose a set of numerical methods and techniques to estimate the effective compressional wave velocities of highly heterogeneous porous rocks along the entire frequency spectrum [1, 2]. To do so, we incorporate the internal structure of the rock at the pore scale and the properties of each of its constituents (density and primary wave velocity). For the low/medium frequency spectrum, we solve the acoustic equation in the frequency domain by the Finite Element Method (FEM), and we postprocess the solution along straight lines to estimate the homogenized compressional wave velocity. To obtain accurate results, we show the necessity to extend the domain by repeating the rock sufficient times with respect to the excitation frequency. Due to this requirement on the computational domain size, we consider non-fitting meshes [3], in which each finite element includes highly-discontinuous material properties. The use of non-fitting meshes allows us to reduce the number of degrees of freedom with respect to the use of traditional conforming fitting meshes. We take advantage of having to repeat the rock when precomputing blocks of the stiffness matrix to reduce the computational cost. At high frequencies, we solve the Eikonal equation by the Fast Marching Method (FMM) [4] to estimate the effective compressional wave velocity. The performance of the proposed methods are illustrated with different numerical experiments over synthetic and real porous rocks where the formations are provided by X-ray micro computed tomography.

References

[1] Á.J. Omella, J. Alvarez-Aramberri, M.Strugaru, V. Darrigrand, D. Pardo, H. González, C. Santos, *A Simulation Method for the Computation of the Effective P-Wave Velocity in Porous Rocks. Part 1: 1D Analysis*, Submitted to Computational Geosciences (2019).

[2] Á.J. Omella, M.Strugaru, J. Alvarez-Aramberri, V. Darrigrand, D. Pardo, H. González, C. Santos, *A Simulation Method for the Computation of the Effective P-Wave Velocity in Porous Rocks. Part 2: 2D and 3D Analysis*, Submitted to Computational Geosciences (2019).

[3] T. Chaumont-Frelet, D. Pardo, Á. Rodríguez-Rozas, *Finite element simulations of logging-while-drilling and extra-deep azimuthal resistivity measurements using non-fitting grids*, Computational Geosciences **22(5)** (2018), 1161-1174.

[4] J.A. Sethian, A.M. Popovici, *3-D traveltime computation using the fast marching method*, Geophysics **64** (1999), 516-523.

SPARSE COMPRESSION OF EXPECTED SOLUTION OPERATORS

Daniel Peterseim
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Joint work with **Michael Feischl**.

We show that the expected solution operator of a prototypical linear elliptic partial differential operator with random diffusion coefficient is well approximated by a computable sparse matrix. This result holds true without structural assumptions on the random coefficient such as stationarity, ergodicity or any characteristic length of correlation. The constructive proof is based localized orthogonal multiresolution decompositions of the solution space for each realization of the random coefficient. The decompositions lead to a block-diagonal representation of the random operator with well-conditioned sparse blocks. Hence, an approximate inversion is achieved by a few steps of some standard iterative solver. The resulting approximate solution operator can be reinterpreted in terms of classical Haar wavelets without loss of sparsity. The expectation of the Haar representation can be computed without difficulty using appropriate sampling techniques. The overall construction leads to a computationally efficient method for the direct approximation of the expected solution operator which is relevant for stochastic homogenization and uncertainty quantification.

References

[1] M. Feischl, D. Peterseim, *Sparse Compression of Expected Solution Operators*, ArXiv e-prints 1807.01741 (2018), 1-18.

ROBUST MODEL REDUCTIONS FOR DARCY EQUATIONS WITH HIGH-CONTRAST COEFFICIENTS

Marcus Sarkis
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Joint work with **Alexandre Madureira**.

Major progress has been made recently to make preconditioners robust with respect to variation of coefficients. A reason for this success is the adaptive selection of primal constraints based on localized generalized eigenvalue problems. In this talk we discuss how to transfer this technique to the field of discretizations. Given a target accuracy, we design a robust model reduction by delocalizing multiscale basis functions and establish a priori energy error estimates with such target accuracy with hidden constants independently of the coefficients.

NONLINEAR FOURIER INTEGRATORS FOR DISPERSIVE EQUATIONS

Katharina Schratz
Heriot-Watt University, UK
Joint work with **Alexander Ostermann and Frédéric Rousset**.

A large toolbox of numerical schemes for nonlinear dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene since the underlying PDEs have very complicated solutions exhibiting high oscillations and loss of regularity. This leads to huge errors, massive computational costs and ultimately provokes the failure of classical schemes. Nevertheless, non-smooth phenomena play a fundamental role in modern physical modeling (e.g., blow-up phenomena, turbulences, high frequencies, low dispersion limits, etc.) which makes it an essential task to develop suitable numerical schemes. In this talk I present a new class of nonlinear Fourier integrators which offer strong geometric structure at low regularity and high oscillations. The key idea in the construction of the new schemes is to tackle and hardwire the underlying structure of resonances into the numerical discretization – linking the finite dimensional discretization to powerful existence results of nonlinear dispersive PDEs in low regularity spaces.

APPROXIMATION OF FOURTH ORDER TWO-POINT SINGULARLY PERTURBED PROBLEMS OF REACTION-DIFFUSION TYPE

Christos Xenophontos
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We consider *fourth order* two-point singularly perturbed problems of reaction-diffusion type and the approximation of their solution by Galerkin's method. We consider both *hp* Finite Elements (FEs) and Isogeometric Analysis (IGA). We first present regularity results which show that the solution may be decomposed into a smooth part, two boundary layers at the endpoints and a (negligible) remainder. Estimates for each part in the decomposition are obtained, which are explicit in the order of differentiation and the singular perturbation parameter [1]. Guided by these results, we construct an approximation using the so-called *Spectral Boundary Layer* mesh in FEs [2] and knot-vector in IGA [3], which converges independently of the singular perturbation parameter. When the error is measured in the energy norm associated with the problem, the convergence rate is exponential, as the degree of the approximating polynomials is increased. Numerical examples illustrating the theory will also be presented.

References

[1] P. Constantinou, *hp Finite Element Methods for Fourth Order Singularly Perturbed Problems*, Ph.D. Dissertation, Department of Mathematics and Statistics, University of Cyprus, 2019.

[2] J.M. Melenk, C. Xenophontos, L. Oberbroeckling, *Robust exponential convergence of hp-FEM for singularly perturbed systems of reaction-diffusion equations with multiple scales*, IMA J. Num. Anal. **33** (2013), 609-628.

[3] K. Liotati, C. Xenophontos, *Isogeometric Analysis for singularly perturbed problems in 1-D: a numerical study*, BAIL2018 Conference (Glascow, Scotland, 2018), 2019 (to appear).