DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D24

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GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D2

DIFFERENTIABILITY OF THE FLOW FOR BV VECTOR FIELDS

Stefano Bianchini
SISSA, Italy

We show that the Regular Lagrangian Flow $X(t, y)$ generated by nearly incompressible BV vector fields admits a derivative $\nabla X(t, y)$ in the sense of measure. This matrix satisfies the ODE

$$\frac{d}{dt}\nabla X(t, y) = (D\mathbf{b}(t))_y \nabla X(t-, y)$$

where $(D\mathbf{b})_y$ is the disintegration of the measure $\int D\mathbf{b}(t)dt$ w.r.t. the trajectories $X(t, y)$.

ON THE LARGE TIME BEHAVIOR OF SOLUTIONS TO BIRTH AND SPREAD TYPE EQUATIONS

Yoshikazu Giga
University of Tokyo, Japan
Joint work with **Hiroyoshi Mitake, Takeshi Ohtsuka, and Hung V. Tran**.

We consider a level-set eikonal-curvature flow equation with an external force. Such a problem is considered as a model to describe an evolution of height of crystal surface by two-dimensional nucleation or possibly some class of growths by screw dislocations. For applications, it is important to estimate growth rate. Without an external source term the solution only spreads horizontally and does not grow vertically so the source term plays a key role for the growth.

Although the large time behavior of parabolic equations are well studied, the equations we study are degenerate parabolic equations where no diffusion effect exists in the normal to each level-set of a solution. Thus, very little is known even for growth rate. Our goal is to describe our recent progress on such type of problems. Ealier results are presented in the paper by H. Mitake, H.V. Tran and the lecturer published in SIAM Math. Anal. in 2016. A review paper is published in Proc. Int. Cong. of Math. in 2018.

In this talk, we first show the existence of asymptotic speed called growth rate. We also discuss asymptotic profile as well as estimates on growth rate.

THE TWO HYPERPLANE CONJECTURE

David Jerison
Massachusetts Institute of Technology, USA

I will introduce a conjecture that I call the *Two Hyperplane Conjecture*, saying that an isoperimetric surface that divides a convex body in half by volume is trapped between parallel hyperplanes. Emanuel Milman has shown that in its strongest, dimension-independent form, my conjecture implies the *Hyperplane Conjecture* of Kannan, Lovász and Simonovits in theoretical computer science, which says that the area of such an isoperimetric surface is comparable, by an absolute constant independent the convex body and its dimension, to the area of some hyperplane dividing the convex body in half. Their conjecture is closely related to several famous unsolved problems in high dimensional convex geometry. But unlike the hyperplane conjecture, the two-hyperplane conjecture has significance even in low dimensions.

I will relate the conjecture to qualitative and quantitative connectivity properties and regularity of area-minimizing surfaces, free boundaries and level sets of eigenfunctions, and report on work in progress with Guy David. The main theme of the talk is that the level sets of least energy solutions to scalar variational problems should be as simple as possible, but no simpler.

ON THE NONLINEAR STABILITY OF BLACK HOLES

Sergiu Klainerman
Princeton University, USA

Black holes are precise mathematical solutions of the Einstein field equations mainly represented by the famous two parameter Kerr family including, as a particular case, the Schwarzschild solution. To correspond to physical reality, i.e. to be more than mathematical artifacts, these solutions have to be stable under small perturbations. While there is today no doubt concerning the physical reality of black holes, based both on observational data and numerical simulations, an actual proof of stability remains a fundamental challenge of Mathematical and Geometric Analysis.

In my talk I will formulate the precise mathematical problem of the nonlinear stability of the Kerr family and describe the main results known so far. In the second part of the talk I will describe my recent result with J. Szeftel "Global Nonlinear Stability of Schwarzschild Spacetime under Polarized perturbations" - arXiv:1711.07597. The result establishes the full nonlinear stability of Schwarzschild spacetime under axially symmetric, polarized perturbations, i.e. stability of solutions of the Einstein vacuum equations for asymptotically flat $1+3$ dimensional Lorentzian metrics which admit a hyper-surface orthogonal space-like Killing vector-field with closed orbits. While building on the remarkable advances made in last 15 years on establishing quantitative linear stability, the paper introduces a series of new ideas among which we emphasize the *general covariant modulation* (GCM) procedure which allows us to construct, dynamically, the center of mass frame of the final state. The mass of the final state itself is tracked using the well known Hawking mass relative to a well adapted foliation itself connected to the center of mass frame. Our work here is the first to prove the nonlinear stability of Schwarzschild in a restricted class of nontrivial perturbations. To a large extent, the restriction to this class of perturbations is only needed to ensure that the final state of evolution is another Schwarzschild space.

ZERO SETS OF LAPLACE EIGENFUCNTIONS

Aleksandr Logunov
Princeton University, USA

We will discuss geometrical and analytic properties of zero sets of harmonic functions and eigenfunctions of the Laplace operator. For harmonic functions on the plane there is an interesting relation between local length of the zero set and the growth of harmonic functions. The larger the zero set is, the faster the growth of harmonic function should be and vice versa. Laplace eigenfunctions on two dimensional sphere are restrictions of homogeneous harmonic polynomials of three variables onto the sphere. Zero sets of such functions are unions of smooth curves with equiangular intersections. Topology of the zero set could be quite complicated, but the total length of the zero set of any spherical harmonic of degree n is comparable to n.

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EFFECTIVE BEHAVIOR OF RANDOM MEDIA

Felix Otto
Max Planck Institute for Mathematics in the Sciences, Germany

In engineering applications, heterogeneous media are often described in statistical terms. This partial knowledge is sufficient to determine the effective, i.e. large-scale behavior. This effective behavior may be inferred from the Representative Volume Element (RVE) method. I report on last years' progress on the quantitative understanding of what is called *stochastic homogenization of elliptic partial differential equations*: optimal error estimates of the RVE method, leading-order characterization of fluctuations, effective multipole expansions. Methods connect to elliptic regularity theory and to concentration of measure arguments.

IMPLICITLY CONSTITUTED FLUID FLOW MODELS: ANALYSIS AND APPROXIMATION

Endre Süli
University of Oxford, UK

Classical models describing the motion of Newtonian fluids, such as water, rely on the assumption that the Cauchy stress is a linear function of the symmetric part of the velocity gradient of the fluid. This assumption leads to the Navier-Stokes equations. It is known however that the framework of classical continuum mechanics, built upon a simple linear constitutive equation for the Cauchy stress, is too narrow to describe inelastic behavior of solid-like materials or viscoelastic properties of materials. Our starting point in this work is therefore a generalization of the classical framework of continuum mechanics, called the implicit constitutive theory, which was proposed recently in a series of papers by Rajagopal. The underlying principle of the implicit constitutive theory in the context of viscous flows is the following: instead of demanding that the Cauchy stress is an explicit (and, in particular, linear) function of the symmetric part of the velocity gradient, one may allow a nonlinear, implicit and not necessarily continuous relationship between these quantities. The resulting general theory therefore admits non-Newtonian fluid flow models with implicit and possibly discontinuous power-law-like rheology.

We develop the analysis of finite element approximations of implicit power-law-like models for viscous incompressible fluids. The Cauchy stress and the symmetric part of the velocity gradient in the class of models under consideration are related by a, possibly multi-valued graph. Using a variety of weak compactness techniques, we show that when the graph of the stress-strain relationship is maximal monotone a subsequence of the sequence of finite element solutions converges to a weak solution of the problem as the discretization parameter, measuring the granularity of the finite element triangulation, tends to zero. When the graph is nonmonotone, a subsequence of the sequence of finite element solutions is shown to converge to a gradient Young-measure solution of the problem. A key new technical tool in the analysis is a finite element counterpart of the Acerbi-Fusco Lipschitz truncation of Sobolev functions. The talk is based on a series of papers with Miroslav Bulíček and Josef Málek (Prague), Miles Caddick (Oxford), Lars Diening (Bielefeld), Christian Kreuzer (Dortmund), and ongoing research with Alexei Gazca-Orozco (Oxford) and Tabea Tscherpel (Aachen).

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GLOBAL ANALYSIS VIA MICROLOCAL TOOLS: FREDHOLM THEORY IN NON-ELLIPTIC SETTINGS

András Vasy
Stanford University, USA
Joint work with **Peter Hintz**.

In this lecture I will describe a microlocal framework for the Fredholm analysis of non-elliptic problems both on manifolds without boundary and manifolds with boundary, introduced in [8] and extended in [6] and various other works. Examples in which such a framework (or a similar framework) has recently been useful include wave propagation on black hole spacetimes, which is the key analytic ingredient for showing the stability of black holes [5, 4], analysis of the resolvent of the generator of the flow for dynamical systems [2], which is the key tool for the analysis of the Ruelle zeta function [1], Feynman propagators in quantum field theory [3, 9] and inverse problems, namely boundary rigidity and tensor tomography [7].

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THE VORTEX FILAMENT EQUATION, THE TALBOT EFFECT, AND NON-CIRCULAR JETS

Luis Vega
University of the Basque Country & BCAM, Spain
Joint work with **Valeria Banica and Francisco De La Hoz**.

We will propose the vortex filament equation as a possible toy model for turbulence, in particular because of its striking similarity to the dynamics of non-circular jets. This similarity implies the existence of some type of Talbot effect due to the interaction of non-linear waves that propagate along the filament. Another consequence of this interaction is the existence of a new class of multi-fractal sets that can be seen as a generalization of the graph of Riemann's non-differentiable function. Theoretical and numerical arguments about the transfer of energy will be also given.

POPULATION DYNAMICS AND CONTROL

Enrique Zuazua
DeustoTech, Bilbao, Basque Country & Universidad Autónoma de Madrid, Spain

Population dynamics is an old subject. Classical models in this field are written in terms of reaction-diffusion equations.

There is a wide literature concerning the dynamical properties of those systems. But much less is known from a control perspective. And control constitutes the ultimate proof of our understanding of a process.

This lecture will be devoted to present two recent results in this area. We first consider a bistable reaction-diffusion arising in the modelling of bilingual populations and then address models involving age structuring and spatial diffusion (of Lotka-McKendrick type).

As we shall see, both aspects require of an in depth understanding of the dynamics of the systems under consideration.

We shall present sharp results on our ability to steer the dynamics of those systems to a prescribed final configuration. Some open problems and future directions of research will also be presented.

This lecture is based on recent joint work in collaboration with D. Maity, C. Pouchol, E. Trélat, M. Tucsnak and J. Zhu.

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TALKS OF SESSION D24

STOCHASTIC INTERFACE MOTION IN THE CAHN-HILLIARD EQUATION

Dirk Blömker
Universität Augsburg, Germany
Joint work with **Dimitra Antonopoulou and Georgia Karali**.

We study the two and three dimensional stochastic Cahn–Hilliard equation in the sharp interface limit. It is given by

$$\partial_t u = -\Delta v + \varepsilon^\sigma \partial_t W, v = -\frac{F'(u)}{\varepsilon} + \varepsilon \Delta u,$$

subject to Neumann boundary conditions on a bounded domain $\mathcal{D}$.

Here $u : \mathcal{D} \times [0,T] \to [-1,1]$ is the scalar concentration field of one of the components in a separation process,for example of binary alloys. The function F is a double well potential, for example with $F'(u) = u - u^3$. The noise is given by a spatially smooth Wiener process W.The small parameter $\varepsilon > 0$ measures the width of transition layers generated during phase separation.

Using formal asymptotic expansions, in [1] we identify the limit. In the case $\sigma = 1$ our results indicate that the stochastic Cahn–Hilliard equation converges to a two-phase Hele-Shaw (or Mullins-Sekerka) problem with stochastic forcing on the transition layers. For the interface

$\Gamma(t) = \{x \in \mathcal{D} \ : \ u(t,x) = 0\}$ in the limit $\varepsilon \to 0$ we obtain

$$\begin{cases} \Delta v = 0 \quad \text{in } \mathcal{D}\backslash\Gamma, \\ \partial_n v = 0 \quad \text{on } \partial\mathcal{D}, \\ v = \lambda H + W \quad \text{on } \Gamma, \\ V = [\partial_n v] \quad \text{on } \Gamma, \end{cases}$$

where H is the mean curvature and V the normal velocity of Γ.The jump term $[\partial_n v]$ denotes the average of the normal derivative of v from both sides of Γ.

In a joint work with S. Yokoyama (Tokyo) we show that the stochastic Hele-Shaw problem has a local smooth solution,given that the initial surface $\Gamma(0)$ is a smooth closed hypersurface that does not touch the boundary of $\mathcal{D}$.

In the case when the noise is sufficiently small (i.e. $\sigma > 1$ sufficiently large), in [1] we can prove rigorously that the limit is a deterministic Hele-Shaw problem without W. The main reason for σ being very large for this result is due to the lack of sufficiently good spectral estimates for the linearized Cahn-Hilliard operator. These are currently only verified in H^{-1}.

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STOCHASTIC NON-ABELIAN YANG-MILLS HEAT FLOW

Ajay Chandra
Imperial College London, UK
Joint work with **Martin Hairer and Hao Shen**.

I will start by describing how a mathematician can think of the problem of constructing a Yang-Mills quantum field theory and how one approach to this problem involves working with singular SPDE. I will then describe results obtained in ongoing joint work with Martin Hairer and Hao Shen regarding local existence and gauge covariance for a singular SPDE that should correspond to a non-Abelian Yang-Mills quantum field theory.

WEAK ERROR EXPANSION FOR MEAN-FIELD SDE

Jean-François Chassagneux
Université Paris Diderot, France
Joint work with **Lukasz Szpruch and Alvin Tse**.

In this work, we study the weak approximation error by particle system of Mean Field SDE. We prove an expansion of this error in terms of the number of particle. Our strategy of proof follows the approach of Talay-Tubaro for weak approximation of SDE by an Euler Scheme. We thus consider a PDE on the Wasserstein space (called the Master Equation in mean-field games literature) and, relying on smoothness properties of the solution, obtain our expansion. We also prove the required smoothness properties under sufficient conditions on the coefficient function.

MEASURE-VALUED DIFFUSION PROCESSES AND MEAN FIELD GAMES

François Delarue
Université Nice-Sophia-Antipolis, France
Joint work with **Alekos Cecchin**.

The purpose of the talk is to address smoothing effect of diffusion processes with values in the space of probability measures, especially when the latter is constructed above a finite set. The motivation comes from the theory of mean field games and of mean field control, which is dedicated to the analysis of equilibria within large population of rational agents and which has been growing fast since the earlier works of Lasry and Lions [3, 4] (see also the recent monographs [1,2]). One key fact is that such equilibria may be described by a stochastic measure-valued process when the whole population is subjected to a common source of noise. In this framework, equilibria turn out to be unique if the common noise induces sufficiently strong regularizing properties onto the space of probability measures. While the latter mostly regards the influence of a common noise onto the equilibria, it also raises interesting questions on the case without common noise: We here show that, by letting the influence of the common

noise tend to zero, we may select, in some cases (known as potential cases), some specific equilibria among all the possible ones.

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ON ROUGH SEMIMARTINGALES

Peter K. Friz

Technische Universität Berlin & WIAS, Germany

Joint work with **K. Le, A. Hocquet, and P. Zorin-Kranich**.

We introduce the new class of rough semimartingales (RSM) with motivation from filtering and SPDE theory. Under natural assumptions, RSM have a unique decomposition. Moreover, RSM are stable under composition with regular functions and stochastic / rough integration. RSM provide further a natural framework to study classes of Markov processes (which are not semimartingales) and we introduce the rough martingale problem.

ASYMPTOTIC FORMULAE IN ROUGH VOLATILITY MODELS

Paul Gassiat
CEREMADE, Université Paris-Dauphine, France
Joint work with **Peter Friz and Paolo Pigato**.

Stochastic volatility models where the volatility behaves similarly to a fractional Brownian motion of Hurst index $H < 1/2$ ("rough volatility") have recently been the subject of considerable interest from the mathematical finance community, due to their ability to reproduce important features observed in market prices. In this talk I will present a result on asymptotics of short-dated call option prices in such models. The proof is based on combining the Laplace method on Wiener space with rough path type techniques.

CYLINDRICAL LÉVY PROCESSES AND LÉVY SPACE-TIME WHITE NOISES

Matthew Griffiths
King's College London, UK
Joint work with **Markus Riedle**.

It is well known (e.g. [1]) that the canonical cylindrical Brownian motion and the Gaussian space-time white noise correspond to each other. In this talk we consider the analogue relation between cylindrical Lévy processes and Lévy space-time white noises. Since there does not exist a "canonical" cylindrical Lévy process the situation is quite different from the Gaussian case. We then apply the established relations by embedding cylindrical Lévy processes in certain Besov spaces, which may be seen as a first result analysing the regular (or irregular) behaviour of the jumps of a cylindrical Lévy process.

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A SCHAUDER-TYCHANOFF TYPE STOCHASTIC FIXPOINT THEOREM AND A COUPLED STOCHASTIC SYSTEM FOR PATTERN FORMATION

Erika Hausenblas
Montanuniversität Leoben, Austria
Joint work with **Mechthild Thalhammer, Tsiry Avisoa Randrianasolo, and Jonas Toelle**.

Mathematical models based on systems of reaction-diffusion equations provide fundamental tools for the description and investigation of various processes in biology, biochemistry, and chemistry; in a specific situation, an appealing characteristic of the arising nonlinear partial differential equations is the formation of patterns, reminiscent of those found in nature. The deterministic Gray–Scott equations constitute an elementary two-component system that describes autocatalytic reaction processes; depending on the choice of the specific parameters, complex patterns of spirals, waves, stripes, or spots appear.

In the derivation of a macroscopic model such as the deterministic Gray–Scott equations from basic physical principles, certain aspects of microscopic dynamics, e.g. fluctuations of molecules, are disregarded; an expedient mathematical approach that accounts for significant microscopic effects relies on the incorporation of stochastic processes and the consideration of stochastic partial differential equations.

In the talk, we first present a stochastic Schauder-Tychanoff type Theorem, then we present as an application the existence of solution of the stochastic Gray-Scott system. Finally, we present some numerical results of the stochastic Gray–Scott equations driven by independent spatially time-homogeneous Wiener processes. The numerical simulations based on the application of a time-adaptive first-order operator splitting method and the fast Fourier transform illustrate the formation of patterns in the deterministic case and their variation under the influence of stochastic noise.

LAGRANGIAN STOCHASTIC MODELS FOR TURBULENT FLOWS AND RELATED PROBLEMS

Jean-François Jabir
National Research University Higher School of Economics, Russia

Lagrangian stochastic models for turbulence define a particular family of Langevin stochastic differential equations, endowing some specific nonlinearities of McKean type, that were originally introduced in the framework of computational fluid dynamics to describe and simulate the motions of a generic particle of a fluid flow. Although these stochastic models are currently applied in various engineering problems, Lagrangian stochastic models for turbulent flows display a certain number of original mathematical problems broadly linked to existence and uniqueness problems for singular McKean-Vlasov dynamics and the validation of related particle approximations; the modeling of boundary conditions for Langevin models; the introduction of distributions constraints ... The first part of the talk will be dedicated to a short presentation of practical interest and the characteristic theoretical problems related to these Lagrangian stochastic models while some resolutions to these problems, in simplified situations, will be discussed in the rest of the talk.

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CONVERGENCE OF GALERKIN APPROXIMATIONS FOR FRACTIONAL STOCHASTIC PDES

Kristin Kirchner
ETH Zürich, Switzerland
Joint work with **David Bolin, Mihály Kovács, and Sonja G. Cox**.

Many models in spatial statistics are based on Gaussian Matérn fields. Motivated by the relation between this class of Gaussian random fields and stochastic partial differential equations (PDEs), we consider the numerical solution of stochastic PDEs with additive spatial white noise on a bounded Euclidean domain $\mathcal{D} \subset \mathbb{R}^d$. The non-local differential operator is given by the fractional power $\mathcal{A}^\beta$, $\beta > 0$, of a second-order elliptic differential operator $\mathcal{A}$.

We propose an approximation which combines recent Galerkin techniques for deterministic fractional-order PDEs with an efficient way to simulate white noise. Under minimal regularity assumptions on the differential operator $\mathcal{A}$, in [1, 2, 3] we perform an error analysis for this approximation showing (i) strong mean-square convergence in $L_2(\mathcal{D})$, (ii) weak convergence, and (iii) convergence in Sobolev spaces: for the approximation of the random field in $L_q(\Omega; H^s(\mathcal{D}))$, where $q \in (0, \infty)$ and $s \in [0, 1]$, as well as for the covariance function of the approximation in the mixed Sobolev space $H^{s,s}(\mathcal{D} \times \mathcal{D})$ at explicit and sharp rates.

For the motivating example of Gaussian Matérn fields, where $\mathcal{A} = -\Delta + \kappa^2$ and $\kappa \equiv$ const., we perform several numerical experiments for various values of the fractional exponent $\beta > 0$ in dimensions $d \in \{1, 2\}$, which attest the theoretical results.

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ASYMPTOTICS FOR LOGISTIC-TYPE EQUATIONS WITH DIRICHLET FRACTIONAL LAPLACIAN

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We will present results on the asymptotics, as $t \to \infty$ and $p \to \infty$, of solutions of the following problem:

$$\frac{\partial u}{\partial t} - (\Delta^{\alpha/2})_{|D} u = au - bu^p, \quad u(0, \cdot) = \varphi. \quad (*)$$

Here D is a bounded Lipschitz domain in $\mathbb{R}^d$, $a > 0, p > 1, \alpha \in (0, 2)$, and b, φ are bounded positive nontrivial Borel functions on D. We show that for suitable a the limit function does not depend on the order of limits and is a unique solution of an obstacle problem.

Equations and systems of type $(*)$ with nonlocal operators appear in many models of population biology. Dirichlet fractional Laplacian in $(*)$ is designed to describe nonlocal dispersal strategy of animals (see [3]). Stationary equations of type $(*)$ have been studied recently in [1] with strictly positive b.

Asymptotics as $p \to \infty$ for solutions to stationary counterpart to $(*)$ with classical Laplacian was studied for the first time in [2]. In this paper it is observed that for large p solutions of this stationary problem behave like solutions of certain steady-state predator-pray models. The methods used in [2] extensively exploit the local character of Dirichlet Laplacian and can not be applied to the case of fractional Laplacian. We present a new method (see [4], [5]) based on the notion of ultracontractivity and probabilistic potential theory. The method we introduce may be applied to a wide class of nonlocal operators.

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BACKWARD EULER-MARUYAMA METHOD FOR SDES WITH MULTIVALUED DRIFT COEFFICIENT

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Joint work with **Monika Eisenmann, Raphael Kruse, and Mihály Kovács**.

We consider the numerical approximation of a multivalued SDE

$$\begin{cases} \mathrm{d}X(t) + f(X(t))\,\mathrm{d}t \ni g(t)\,\mathrm{d}W(t), & t \in (0,T], \\ X(0) = X_0, \end{cases}$$

where the mapping $f\colon \mathbf{R}^d \to 2^{\mathbf{R}^d}$ is maximal monotone, of at most polynomial growth, coercive, and fulfills the condition

$$\langle f_v - f_z, z - w \rangle \leq \langle f_v - f_w, v - w \rangle,$$

for every $v, w, z \in \mathbf{R}^d$, $f_v \in f(v)$, $f_w \in f(w)$, and $f_z \in f(z)$ as proposed in [1]. Under these low regularity assumptions on the drift coefficient, we can prove well definedness of the backward Euler method, as well as the strong convergence with a rate of $\frac{1}{4}$, if g lies in a suitable Hölder space.

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AVERAGING DYNAMICS DRIVEN BY FRACTIONAL NOISE

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Joint work with **Martin Hairer**.

Two scale stochastic equations model evolutions of random motions under the influence of fast motions. An overwhelming amount of efforts have been devoted to where the fast motion is assumed to have a Markov property or have independent increments, time series data begs to differ. I hope to explain the effective dynamics for a slow/fast systems where the slow system is driven by a fractional Brownian motion, and perhaps also to touch on further developments.

DYNAMICAL LOW RANK APPROXIMATION OF RANDOM TIME DEPENDENT PDES

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Joint work with **Yoshihito Kazashi and Eva Vidlicková**.

Partial differential equations with stochastic coefficients and input data arise in many applications in which the data of the PDE need to be described in terms of random variables/fields due either to a lack of knowledge of the system or to its inherent variability. The numerical approximation of statistics of the solution poses several challenges when the number of random parameters is large and/or the parameter-to-solution map is complex, and effective surrogate or reduced models are of great need in this context.

In this talk we consider time dependent PDEs with few random parameters and seek for an approximate solution in separable form that can be written at each time instant as a linear combination of linearly independent spatial functions multiplied by linearly independent random variables (low rank approximation) in the spirit of a truncated Karhunen-Loève expansion. Since the optimal deterministic and stochastic modes can significantly change over time, we consider here a dynamical approach where those modes are computed on the fly as solutions of suitable evolution equations. From a geometrical point of view, this corresponds to constraining the original dynamics to the manifold of fixed rank functions, i.e. functions that can be written in separable form with a fixed number of terms. Equivalently, the original equations are projected onto the tangent space to the manifold of fixed rank functions along the approximate trajectory, similarly to the Dirac-Frenkel variational principle in quantum mechanics.

We discuss the construction of the method, present an existence result for a random semi-linear evolutionary equation, and discuss practical numerical aspects for several time dependent PDEs with random parameters, including the heat equation with a random diffusion coefficient; the incompressible Navier-Stokes equations with random Dirichlet boundary conditions; the wave equation with random wave speed.

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EXISTENCE AND UNIQUENESS OF STRONG SOLUTIONS OF SINGULAR SDES

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Joint work with **Salah Mohammed and Ludovic Tangpi**.

In this talk, we are interested in existence and uniqueness of strong solutions for stochastic differential equations with irregular drift coefficients. The driving noise is a d-dimensional Brownian motion. The method relies on Malliavin calculus and as a byproduct, we obtain Malliavin differentiability of the solutions. Existence of Sobolev differentiable flows in small time will also be discussed.

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LONG TIME BEHAVIOUR, PHASE TRANSITIONS AND FLUCTUATIONS FOR THE McKEAN-VLASOV EQUATION

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Joint work with **José A. Carrillo, Matias Gonzalo Delgadino, Susana N. Gomes, Rishabh S. Gvalani, and André Schlichting**.

We study the long time behaviour, the number and structure of stationary solutions and fluctuations for the McKean-Vlasov equation, a nonlinear nonlocal Fokker-Planck type equation that describes the mean field limit of a system of weakly interacting diffusions. We consider two cases: the McKean-Vlasov equation in a multiscale confining potential with quadratic, Curie-Weiss, interaction (the so-called Dasai-Zwanzig model), and the McKean-Vlasov dynamics on the torus with periodic boundary conditions and with a localized interaction. Our main objectives are the study of convergence to a stationary state, the construction of the bifurcation diagram for the stationary problem and the study of fluctuations around the McKean-Vlasov limit, in particular past the phase transition. The application of our work to the study of models for opinion formation and of synchronization for Kuramoto-type models is also discussed. This talk is based on the recent works [1, 2].

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A ROUGH SUPERBROWNIAN MOTION

Nicolas Perkowski
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Joint work with **Tommaso Cornelis Rosati**.

We consider a 1- or 2-dimensional branching random walk in a small random potential and show that its large scale behavior is described by a new stochastic process, which we call rough superBrownian motion. This process is a mixture of the classical superBrownian motion and the continuous parabolic Anderson model (PAM), where the superBrownian part captures fluctuations caused by the branching mechanism and the PAM part describes the large scale behavior of the random potential. We use pathwise arguments to deal with the PAM-singularity, and martingale tools to deal with the singularity from the superBrownian part. We also study the survival properties of the rough superBrownian motion and show that it behaves very differently from its classical counterpart.

A SUPPORT THEOREM FOR SINGULAR STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

Philipp Schoenbauer
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Joint work with **Martin Hairer**.

The purpose of this talk is to present a far-reaching generalization of the support theorem of Stroock and Varadhan. Recall that, given a stochastic differential equation (SDE), this theorem considers all ordinary differential equations (ODEs) which formally look the same as the SDE, but with the noise replaced by an arbitrary smooth function. The support of the SDE is then shown to be the closure of the set of all solutions to these ODEs. We prove a support theorem in the same spirit for stochastic partial differential equations (SPDEs). As part of our analysis we establish the "correct" way to deal with the issue of divergent renormalisation constants in such a description. (This issue makes it difficult to even guess the correct formulation of a support theorem.) Our approach applies to a range of interesting (singular) SPDEs, among them

the stochastic quantization equations and the generalised KPZ equations. As an important corollary, we show the uniqueness of the invariant measure for the 3D stochastic quantization equation.

LANGEVIN DYNAMICS DERIVED FROM QUANTUM MECHANICS AND ITS RELATION TO STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

Anders Szepessy
Royal Institute of Technology (KTH), Sweden
Joint work with **Håkon Hoel**.

In the study of stochastic partial differential equation one may wonder what is the noise? Often the stochasticity modelled in partial differential equations has its origin in thermal fluctuations.

Starting from a quantum formulation of a molecular system coupled to a heat bath, I will show that ab initio Langevin dynamics, with a certain rank one friction matrix determined by the coupling, approximates the quantum system more accurately than any Hamiltonian system, for large mass ratio between the system and heat bath nuclei.

will also give an example of course-graining a stochastic molecular dynamics equation to obtain a continuum stochastic partial differential equation for phase transitions.

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MEAN-FIELD LANGEVIN DYNAMICS AND ENERGY LANDSCAPE OF NEURAL NETWORKS

Lukasz Szpruch
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In this paper we present a probabilistic analysis of the long-time behaviour of the nonlocal, diffusive equations with a gradient flow structure in 2-Wasserstein metric. Our work is motivated by a desire to provide a theoretical underpinning for the convergence of stochastic gradient type algorithms widely used for non-convex learning tasks such as training of deep neural networks. The key insight is that the certain class of the finite dimensional non-convex problems becomes convex when lifted to infinite dimensional space of measures. We leverage this observation and show that the flow of marginal laws induced by the Mean Field Langevin Dynamics equation converges to the stationary distribution which is exactly the minimiser of the energy functional. At the heart of our analysis is a pathwise perspective on Otto calculus used in gradient flow literature which is of independent interest. Our proof of convergence to stationary probability measure is novel and it relies on a generalisation of LaSalle's invariance principle.

MAGNUS-TYPE INTEGRATOR FOR THE FINITE ELEMENT DISCRETIZATION OF SEMILINEAR PARABOLIC NON-AUTONOMOUS SPDEs DRIVEN BY MULTIPLICATIVE NOISE

Antoine Tambue
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Joint work with **Jean Daniel Mukam**.

This paper aims to investigate numerical approximation of a general second order non-autonomous semilinear parabolic stochastic partial differential equation (SPDE) driven by multiplicative

noise. Numerical approximations of autonomous SPDEs are thoroughly investigated in the literature, while the non-autonomous case is not yet understood. We discretize the non-autonomous SPDE driven by multiplicative noise by the finite element method in space and the Magnus-type integrator in time. We provide a strong convergence proof of the fully discrete scheme toward the mild solution in the root-mean-square L^2 norm. The result reveals how the convergence orders in both space and time depend on the regularity of the noise and the initial data. In particular, for multiplicative trace class noise we achieve convergence order $\mathcal{O}\left(h^2\left(1+\max(0,\ln\left(t_m/h^2\right)\right)+\Delta t^{1/2}\right)$. Numerical simulations to illustrate our theoretical finding are provided.

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A-PRIORI BOUNDS FOR SINGULAR SPDES

Hendrik Weber
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Joint work with **Ajay Chandra and Augustin Moinat**.

The theory of regularity structures is a powerful tool to develop a stable solution theory for a whole class of stochastic PDEs arising in statistical mechanics and quantum field theory. Initiated in Hairer's groundbreaking work in 2013, in only a few years an astonishingly general solution theory covering essentially all equations which satisfy a certain scaling condition (subcriticality or super-renormalizability), has been developed. However, up to now, most results only gave control over solutions for *small times* and on *bounded* spatial domains.

The aim if this talk is to present a method to prove a priori estimates in the framework regularity structures. These bounds complement the short time existence and uniqueness theory to obtain control of solutions globally in time and on unbounded domains. Our bounds are implemented in the example of the dynamic Φ^4 equation, which is formally given by

$$(\partial_t - \Delta)u = -u^3 + \infty u + \xi.$$

This equation is subcritical if the distribution ξ is of class $C^{-3+\frac{\delta}{2}}$ for $\delta > 0$, and we obtain bounds for all such ξ. An analogy to the regularity of white noise suggests to interpret this as a solution theory for Φ^4 in dimension $4 - \delta$.

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LARGE DEVIATION PRINCIPLES FOR FIRST-ORDER SCALAR CONSERVATION LAWS WITH STOCHASTIC FORCING

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Joint work with **Zhao Dong, Jianglun Wu, and Rangrang Zhang**.

In this paper, we established the Freidlin-Wentzell type large deviation principles for first-order scalar conservation laws perturbed by small multiplicative noise. Due to the lack of the viscous terms in the stochastic equations, the kinetic solution to the Cauchy problem for these first-order conservation laws is studied. Then, based on the well-posedness of the kinetic solutions, we show that the large deviations holds by utilising the weak convergence approach.

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RANDOM PERIODICITY: THEORY AND MODELLING

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Random periodicity is ubiquitous in the real world. In this talk, I will provide the concepts of random periodic paths and periodic measures to mathematically describe random periodicity. It is proved that these two different notions are "equivalent". Existence and uniqueness of random periodic paths and periodic measures for certain stochastic differential equations are proved. An ergodic theory is established. It is proved that for a Markovian random dynamical system, in the random periodic case, the infinitesimal generator of its Markovian semigroup has infinite number of equally placed simple eigenvalues including 0 on the imaginary axis. This is in contrast to the mixing stationary case in which the Koopman-von Neumann Theorem says there is only one eigenvalue 0, which is simple, on the imaginary axis. Geometric ergodicity for some stochastic systems is obtained. Possible applications e.g. in stochastic resonance will be discussed.

This talk is based on a series of work with Chunrong Feng et al.