DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS SESSION D31

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D3

KAM THEORY FOR SECONDARY TORI

Luigi Chierchia
Roma Tre University, Italy
Joint work with **Luca Biasco**.

As well known, classical KAM (Kolmogorov, Arnold, Moser) theory deals with the persistence, under small perturbations, of real-analytic (or smooth) Lagrangian tori for nearly-integrable non-degenerate Hamiltonian systems. In this talk I will present a new *uniform* KAM theory apt to deal also with secondary tori, i.e., maximal invariant tori (with different homotopy) "generated" by the perturbation (and that do not exist in the integrable limit). The word "uniform" means that primary and secondary tori are constructed simultaneously; in particular, in the case of Newtonian mechanical systems on $\mathbf{T}^d$, it is proven that, for generic perturbations, the union of primary and secondary tori leave out a region of order $\varepsilon|\log\varepsilon|^a$, if ε is the norm of the perturbation, in agreement (up to the logarithmic correction) with a conjecture by Arnold, Kozlov and Neishtadt.

Some of these results have been announced in the note [1].

References

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SOME GEOMETRIC MECHANISMS FOR ARNOLD DIFFUSION

Rafael de la Llave
Georgia Institute of Technology, USA

We consider the problem whether small perturbations of integrable mechanical systems can have very large effects. Since the work of Arnold in 1964, it is known that there are situations where the perturbations can accumulate. This can be understood by noting that the small perturbations generate some invariant structures that, with their stable and unstable manifolds can cover a large region in phase space. We will present recent developments in identifying these invariant objects, both in finite and in infinite dimensions.

DIFFERENTIAL EQUATIONS FOR NETWORK CENTRALITY

Desmond Higham
University of Edinburgh, UK

I will derive and discuss two circumstances where ODEs arise in the study of large, complex networks. In both cases, the overall aim is to identify the most important nodes in a network-this task is useful, for example, in digital marketing, security and epidemiology. In one case, we define our node centrality measure using the concept of nonbacktracking walks. This requires us to derive an expression for an exponential-type generating function associated with the walk counts of different length. Solving the ODE leads to a computationally useful characterisation of the centrality measure. In another case, we are presented with a time-ordered sequence of networks; for example, recording who emailed who over each one-minute time-window. Here, by considering the asymptotic limit as the window size tends to zero, we arrive at a limiting ODE that may be treated with a numerical method. Results for both algorithms will be illustrated on real network examples.

ROBUST CHAOS: A TALE OF BLENDERS, THEIR COMPUTATION, AND THEIR DESTRUCTION

Hinke Osinga
University of Auckland, New Zealand
Joint work with **Stephanie Hittmeyer, Bernd Krauskopf, and Katsutoshi Shinohara**.

A blender is an intricate geometric structure of a three- or higher-dimensional diffeomorphism [1]. Its characterising feature is that its invariant manifolds behave as geometric objects of a dimension that is larger than expected from the dimensions of the manifolds themselves. We introduce a family of three-dimensional Hénon-like maps and study how it gives rise to an explicit example of a blender [2, 3]. We employ our advanced numerical techniques to present images of blenders and their associated one-dimensional stable manifolds. Moreover, we develop an effective and accurate numerical test to verify what we call the *carpet property* of a blender. This approach provides strong numerical evidence for the existence of the blender over a large parameter range, as well as its disappearance and geometric properties beyond this range. We conclude with a discussion of the relevance of the carpet property for chaotic attractors.

References

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THE JOINT SPECTRAL RADIUS AND FUNCTIONAL EQUATIONS: A RECENT PROGRESS

Vladimir Protasov
University of L'Aquila, Italy & Lomonosov Moscow State University, Russia

Joint spectral radius of matrices have been used since late eighties as a measure of stability of linear switching dynamical systems. Nearly in the same time it has found important applications in the theory of refinement equations (linear difference equations with a contraction of the argument), which is a key tool in the construction of compactly supported wavelets and of subdivision schemes in approximation theory and design of curves and surfaces. However, the computation or even estimation of the joint spectral radius is a hard problem. It was shown by Blondel and Tsitsiklis that this problem is in general algorithmically undecidable. Nevertheless recent geometrical methods [1,2,3,4] make it possible to efficiently estimate this value or even find it precisely for the vast majority of matrices. We discuss this issue and formulate some open problems.

References

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OPTIMAL CONTROL AND APPLICATIONS TO AEROSPACE

Emmanuel Trélat
Sorbonne Université, France

I will report on nonlinear optimal control theory and show how it can be used to address problems in aerospace, such as orbit transfer. The knowledge resulting from the Pontryagin maximum principle is in general insufficient for solving adequately the problem, in particular due to the difficulty of initializing the shooting method. I will show how the shooting method can be successfully combined with numerical homotopies, which consist of deforming continuously a problem towards a simpler one. In view of designing low-cost interplanetary space missions, optimal control can also be combined with dynamical system theory, using the nice dynamical properties around Lagrange points that are of great interest for mission design.

SMALL DIVISORS AND NORMAL FORMS

Warwick Tucker
Uppsala University, Sweden
Joint work with **Zbigniew Galias**.

In this talk, we will discuss the computational challenges of computing trajectories of a non-linear ODE in a region close to a saddle-type fixed-point. By introducing a carefully selected close to identity change of variables, we can bring the non-linear ODE into an "almost" linear system. This normal form system has an explicit transfer-map that transports trajectories away from the fixed point in a controlled manner. Determining the domain of existence for such a change of variables poses some interesting computational challenges. The proposed method is quite general, and can be extended to the complex setting with spiral saddles. It is also completely constructive which makes it suitable for practical use. We illustrate the use of the method by a few examples.

ORTHOGONAL POLYNOMIALS AND PAINLEVÉ EQUATIONS

Walter Van Assche
KU Leuven, Belgium

Painlevé equations are nonlinear differential equations for which the branch points do not depend on the initialcondition (no movable branch points). There are also discrete Painlevé equations which are non-linearrecurrence relations with enough structure (symmetry and geometry) that make them integrable.Both the discrete and continuous Painlevé equations appear in a natural way in the theory of orthogonalpolynomials. The recurrence coefficients of certain families of orthogonal polynomials often satisfya discrete Painlevé equation. The Toda equations describing the movement of particles with an exponentialinteraction with their neighbors, is equivalent to an exponential modification $e^{xt}\, d\mu(x)$ of the orthogonality measure $d\mu$ for a family of orthogonal polynomials, and the corresponding recurrencecoefficients satisfy the Toda equations, which is a system of differential-difference equations. Combining this with the discrete Painlevé equations then gives a Painlevé differential equation.We will illustrate this by a number of examples. The relevant solutions of these Painlevé equationsare usually in terms of known special functions, such as the Bessel functions, the Airy function, parabolic cylinder functions, or (confluent) hypergeometric functions.

References

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DELAY EQUATIONS AND TWIN SEMIGROUPS

Sjoerd Verduyn Lunel
Utrecht University, Netherlands
Joint work with **Odo Diekmann**.

A delay equation is a rule for extending a function of time towards the future on the basis of the (assumed to be) known past. By translation along the extended function (i.e., by updating the history), one defines a dynamical system. If one chooses as state-space the continuous initial functions, the translation semigroup is continuous, but the initial data corresponding to the fundamental solution is not contained in the state space.

In ongoing joint work with Odo Diekmann, we choose as state space the space of bounded Borel functions and thus sacrifice strong continuity in order to gain a simple description of the variation-of-constants formula.

The aim of the lecture is to introduce the perturbation theory framework of twin semigroups on a norming dual pair of spaces, to show how renewal equations fit in this framework and to sketch how neutral equations can be covered. The growth of an age-structured population serves as a pedagogical example.

DYNAMICAL SYSTEM APPROACH TO SPECTRAL THEORY OF QUASI-PERIODIC SCHRÖDINGER OPERATORS

Jiangong You
Nankai University, China

The spectral theory of quasiperiodic operators is a fascinating field which continuously attracts a lot of attentions for its rich background in quantum physics as well as its rich connections with many mathematical theories and methods. In this talk, I will briefly introduce the problems in this field and their connections with dynamical system. I will also talk about some recent results joint with Avila, Ge, Leguil, Zhao and Zhou on both spectrum and spectral measure by reducibility theory in dynamical systems.

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TALKS OF SESSION D31

NEW TRENDS ON GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS

Everaldo Bonotto
University of São Paulo, Brazil
Joint work with **Marcia Federson and Rodolfo Collegari**.

The theory of generalized ordinary differential equations lies on the fact that these equations encompass various types of other equations such as ordinary differential equations (ODEs), impulsive differential equations (IDEs), measure differential equations (MDEs), functional differential equations and dynamic equations on time scales. Moreover, the theory of generalized ordinary differential equations deals with problems, especially, when the functions involved have many discontinuities and/or are of unbounded variation.

In this talk, we present the general theory of generalized ordinary differential equations and also the most recent results on this topic. In special, we show that this theory also encompass the stochastic differential equations.

References

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GENERALIZED PERIODIC SOLUTIONS TO PERTURBED KEPLER PROBLEMS

Alberto Boscaggin
University of Turin, Italy
Joint work with **Walter Dambrosio, Duccio Papini, Rafael Ortega, and Lei Zhao**.

For the perturbed Kepler problem

$$\ddot{x} = -\frac{x}{|x|^3} + \nabla_x W(t,x), \qquad x \in \mathbb{R}^d,$$

with $d = 2$ or $d = 3$, we discuss the existence of periodic solutions, possibly interacting with the singular set $(x = 0)$. First, a suitable notion of *generalized* solution is introduced, based on the theory of regularization of collisions in Celestial Mechanics; second, existence and multiplicity results are provided, with suitable assumptions on the perturbation term W, by the use of sympletic and variational methods.

References

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[2] A. Boscaggin, R. Ortega, L. Zhao, *Periodic solutions and regularization of a Kepler problem with time-dependent perturbation*, Trans. Amer. Math. Soc., to appear.

GLOBAL DYNAMICS FOR NICHOLSON'S BLOWFLIES SYSTEMS

Teresa Faria
University of Lisbon, Portugal

We study the global asymptotic behaviour of solutions for a Nicholson's blowflies system with patch structure and multiple discrete delays:

$$x_i'(t) = -d_i(t)x_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)x_j(t) + \sum_{k=1}^{m} \beta_{ik}(t)x_i(t-\tau_{ik}(t))e^{-c_i(t)x_i(t-\tau_{ik}(t))}, \quad i = 1, \dots, n,\ t \geq 0, \tag{1}$$

where all the coefficient and delay functions are continuous, nonnegative and bounded, $d_i(t) > 0, c_i(t) \geq c_i > 0$ and $\beta_i(t) := \sum_{k=1}^{m} \beta_{ik}(t) > 0$ for $t \geq 0$, $i, j = 1, \dots, n, k = 1, \dots, m$. For the autonomous version of (1), an overview of results concerning the total or partial extinction of the populations, uniform persistence, existence and absolute global asymptotic stability of a positive equilibrium is presented, see [3, 4]. A criterion for the global attractivity of the positive equilibrium depending on the size of delays is also given [2], extending results in [1]. Most of these results rely on some properties of the so-called community matrix and on the specific shape of the nonlinearity.

For non-autonomous systems (1), sufficient conditions for both the extinction of the populations in all the patches and the permanence of the system were established in [3]. In this case, (1) is treated as a perturbation of the linear homogeneous cooperative ODE system $x_i'(t) = -d_i(t)x_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)x_j(t)$ $(1 \leq i \leq n)$, for which conditions for its asymptotic stability are imposed; although the nonlinearities in (1) are non-monotone, techniques of cooperative DDEs are used.

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PARABOLIC ARCS FOR TIME-DEPENDENT PERTURBATIONS OF THE KEPLER PROBLEM

Guglielmo Feltrin
University of Udine, Italy
Joint work with **Alberto Boscaggin, Walter Dambrosio, and Susanna Terracini**.

We prove the existence of parabolic arcs with prescribed asymptotic direction for the equation

$$\ddot{x} = -\frac{x}{|x|^3} + \nabla W(t,x), \qquad x \in \mathbb{R}^d,$$

where $d \geq 2$ and W is a (possibly time-dependent) lower order term, for $|x| \to +\infty$, with respect to the Kepler potential $1/|x|$. The result applies to the elliptic restricted three-body problem and, more generally, to the restricted $(N+1)$-body problem. The proof relies on a perturbative argument, after an appropriate formulation of the problem in a suitable functional space.

ASPECTS OF SPECIAL FUNCTIONS

Galina Filipuk
University of Warsaw, Poland

Special functions often solve ordinary differential equations. The well-known hypergeometric and Heun functions solve linear second order differential equations, whereas the Painlevé transcendents solve nonlinear second order differential equations. In this talk I shall overview some aspects of linear and nonlinear special functions and their differential equations. I shall also describe connections of linear equations to Okubo type systems. <p/>

References

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[2] G. Filipuk, A. Ishkhanyan, J. Dereziński, *On the Heun equation (tentative title)*, Preprint 2019.

EXISTENCE AND MULTIPLICITY RESULTS FOR SYSTEMS OF FIRST ORDER DIFFERENTIAL EQUATIONS VIA THE METHOD OF SOLUTION-REGIONS

Marlène Frigon
Université de Montréal, Canada

We present existence and multiplicity results for systems of first order differential equations of the form:

$$\begin{aligned} &u'(t) = f(t, u(t)) \quad \text{for a.e. } t \in [0, T], \\ &u \in \mathcal{B}; \end{aligned} \tag{1}$$

where $f : [0, T] \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function and $\mathcal{B}$ denotes a boundary value condition. No growth conditions will be imposed on f. Even though this problem was widely treated, few multiplicity results can be found in the literature.

In the case where there is only one equation ($N = 1$), the method of upper and lower solutions is well known and very useful to obtain existence and multiplicity results. In particular, this was done in the pioneering work of Mawhin [4].

Very few multiplicity results were obtained in the case where the system (1) has more than one equation ($N > 1$). In [3], Frigon and Lotfipour introduced the notion of strict solution-tubes on which rely their multiplicity results. This method was used in [1] to obtain multiplicity results for systems of differential equations with a nonlinear differential operator.

We will present the method of solution-regions to establish existence and multiplicity results for the system (1). A solution-region will be a suitable set R in $[0, T] \times \mathbb{R}^N$ for which we will deduce that it contains the graph of viable solutions. We will show that this method generalizes the methods of upper and lower solutions and of solution-tubes. We will introduce also the notion of strict solution-regions and we will give conditions insuring the existence of at least three viable solutions of (1). Many non trivial examples will be presented throughout this presentation to show that the method of solution-regions is a powerful tool to establish the existence of solutions of systems of differential equations.

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PERIODIC SOLUTIONS OF THE BRILLOUIN ELECTRON BEAM FOCUSING EQUATION: SOME RECENT RESULTS

Maurizio Garrione
Polytechnic University of Milan, Italy
Joint work with **Roberto Castelli and Manuel Zamora**.

We present some recent results dealing with the existence of (positive) 2π-periodic solutions of the Brillouin electron beam focusing equation

$$\ddot{x} + b(1+\cos t)x = \frac{1}{x}, \tag{1}$$

in dependence on the real parameter b. In literature, such a study was particularly stimulated by a conjecture that arose some decades ago, saying that for every $b \in (0, 1/4)$ there exists a 2π-periodic solution of (1). So far, the conjecture was neither shown to be true nor disproved, but the existence results were improved and refined step by step, reaching existence for $b \in (0, b_0]$, with $b_0 \approx 0.1645$, having a strong relation with the first stability interval of the associated Mathieu equation $\ddot{x} + b(1 + \cos t)x = 0$. In this talk, we will try to make a little step further in understanding the picture for the 2π-periodic solvability of (1). On the one hand, we will see that existence may hold true also for values of b belonging to stability intervals of the Mathieu equation other than the first, explicitly exhibiting one of such intervals. On the other hand, we will show that there exists $b^* \approx 0.248$ such that existence holds for $b \in (0, b^*]$ and for $b = b^*$ the branch of solutions obtained through symmetry extension of Neumann ones undergoes a fold bifurcation, so that, as a by-product, multiplicity of 2π-periodic solutions for b close to b^* is proved. This raises further questions about (1) and the validity of the related conjecture. The techniques used rely, respectively, on careful winding number estimates and computer-assisted proofs.

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DYNAMICS ON LATTICES

Hermen Jan Hupkes
University of Leiden, Netherlands

We study dynamical systems posed on lattices, with a special focus on the behaviour of basic objects such as travelling corners, expanding spheres and travelling waves. Such systems arise naturally in many applications where the underlying spatial domain has a discrete structure. Think for example of the propagation of electrical signals through nerve fibres, where the the myeline coating has gaps at regular intervals. Or the study of magnetic spins arranged on crystal lattices.

Throughout the talk we will explore the impact that the spatial topology of the lattice has on the dynamical behaviour of solutions. We will discuss lattice impurities, the consequences of anistropy and make connections with the field of crystallography.

FINITE CYCLICITY OF THE CONTACT POINT IN SLOW-FAST INTEGRABLE SYSTEMS OF DARBOUX TYPE

Renato Huzak
Hasselt University, Belgium

Using singular perturbation theory and family blow-up we prove that nilpotent contact points in deformations of slow-fast Darboux integrable systems have finite cyclicity. The deformations are smooth or analytic depending on the region in the parameter space. This paper is a natural continuation of [M. Bobieński, P. Mardesic and D. Novikov, 2013] and [M. Bobieński and L. Gavrilov, 2016] where one studies limit cycles in polynomial deformations of slow-fast Darboux integrable systems, around the ”integrable” direction in the parameter space. We extend the existing finite cyclicity result of the contact point to analytic deformations, and under some assumptions we prove that the contact point has finite cyclicity around the ”slow-fast” direction in the parameter space.

BIFURCATION OF EQUILIBRIUM FORMS OF AN ELASTIC ROD ON A TWO-PARAMETER WINKLER FOUNDATION

Joanna Janczewska
Gdańsk University of Technology, Poland
Joint work with **Marek Izydorek, Nils Waterstraat, and Anita Zgorzelska**.

Bifurcation theory is one of the most powerful tools in studying deformations of elastic rods, plates and shells. Numerous works have been devoted to the study of bifurcation in elasticity theory. We consider two-parameter bifurcation of equilibrium states of an elastic rod on a deformable foundation of Winkler's type. The rod is being compressed by forces at the ends. The left end is free, but we require the shear force to vanish. At the right end, we assume the rod is simply supported. Our main theorem shows that bifurcation occurs if and only if the linearization of our problem has nontrivial solutions. In fact our proof, based on the concept of the Brouwer degree, gives more, namely that from each bifurcation point there branches off a continuum of solutions.

References

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ON THE MARKUS-NEUMANN THEOREM

Víctor Jiménez López
University of Murcia, Spain
Joint work with **José Ginés Espín Buendía**.

According to a well-known result by L. Markus [2], extended by D.A. Neumann in [3], two continuous surface flows are equivalent if and only if there is a homeomorphism preserving orbits and time directions of their separatrix configurations. In this talk, based on the paper [1], some examples are shown to illustrate that the Markus-Neumann theorem, as stated in the original papers, needs not work. Also, we show how the (nontrivial) gap of the proof can be amended to get a correct (and somewhat more general) version of the theorem.

References

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ODES AND GEOMETRIC STRUCTURES ON SOLUTION SPACES

Wojciech Kryński
Polish Academy of Sciences, Poland

We shall consider ordinary differential equations (ODEs) from the geometric vewpoint. Our aim is study geometric structures appearing on the solution spces to ODEs. In particular, for the third order ODEs one can get canonical conformal structures on the solution spaces.

Higher order generalizations lead to $GL(2)$-geometry and, in general, to the so-called causal or cone geometry. In the talk, we shall also present applications to the twistor theory and to the integrable systems of partial differential equaitons.

BIFURCATION OF A MEAN CURVATURE PROBLEM IN MINKOWSKI SPACE ON AN EXTERIOR DOMAIN

Yong-Hoon Lee
Pusan National University, South Korea
Joint work with **Rui Yang**.

We study the existence of positive radial solutions for a mean curvature problem in Minkowski space on an exterior domain. Based on C^1-regularity of solutions, which is closely related to the property of nonlinearity f near 0, we make use of the global bifurcation theory to establish some existence results of positive radial solutions when f is sublinear at ∞.

BLOWFLY EQUATIONS: HISTORY, CURRENT RESEARCH AND OPEN PROBLEMS

Gergely Röst
University of Szeged, Hungary & University of Oxford, UK

The nonlinear delay differential equation today known as Nicholson's blowfly equation was introduced in 1980 to offer an explanation for a curious dataset that had been found in experiments with a laboratory insect population. Complex dynamics arises due to the interplay of the time delay and a non-monotone feedback.

In addition to being an elegant biological application, this equation has inspired the development of a large number of analytical and topological tools for infinite dimensional dynamical systems, including local and global Hopf-bifurcation analysis for delay differential equations, asymptotic analysis, stability criteria, invariant manifolds, singular perturbation techniques, invariance principles, order preserving semiflows by non-standard cones in Banach spaces, and the study of slowly and rapidly oscillatory solutions.

In this talk we give an overview of these developments, and discuss three current research directions, namely

(i) a more refined model of age-dependent intraspecific competition in pre-adult life stages and its effects on adult population dynamics;

(ii) the effect of environmental heterogeneity on nonlinear oscillations;

(iii) the evolution of maturation periods.

ON THE STUDY OF POSITIVE SOLUTIONS FOR SEMIPOSITONE p-LAPLACIAN PROBLEMS

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Joint work with **Lee, Shivaji, and Son**.

In this talk, I introduce what semipositone problems are and some issues on it. I focus on constructions of a subsolution to show the existence of positive solutions for sublinear (infinite) semipositone problems on bounded domains. Moreover, I discuss the existence and uniqueness of positive radial solutions for sublinear (infinite) semipositone p-Laplacian problems on the exterior of a ball with nonlinear boundary conditions. This talk is mainly based on joint works with Lee, Shivaji and Son.

ANALYTIC PROPERTIES OF THE COMPLETE FORMAL NORMAL FORM FOR THE BOGDANOV–TAKENS SINGULARITY

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Joint work with **Henryk Żołądek**.

In [19] a complete formal normal forms for germs of 2-dimensional holomorphic vector fields with nilpotent singularity was obtained. That classification is quite nontrivial (7 cases), but it can be divided into general types like in the case of the elementary singularities. One could expect that also the analytic properties of the normal forms for the nilpotent singularities are analogous to the case of the elementary singularities. This is really true. In the cases analogous to the focus and the node the normal form is analytic. In the case analogous to the nonresonant saddle the normal form is often nonanalytic due to the small divisors phenomenon. In the cases analogous to the resonant saddles (including saddle-nodes) the normal form is nonanalytic due to bad properties of some homological operators associated with the first nontrivial term in the orbital normal form.

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ON THE UNIQUENESS OF POSITIVE RADIAL SOLUTIONS OF SUPERLINEAR ELLIPTIC EQUATIONS IN ANNULI

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Joint work with **Naoki Shioji and Kohtaro Watanabe**.

The Dirichlet problem

$$\begin{cases} \Delta u + f(u) = 0 & \text{in } x \in A, \\ \quad u = 0 & \text{on } \partial A, \end{cases}$$

is considered, where $A := \{x \in \mathbb{R}^N : a < |x| < b\}$, $N \in \mathbb{N}$, $N \geq 2$, $0 < a < b < \infty$, $f \in C^1[0, \infty)$, $f(u) > 0$ and $uf'(u) \geq f(u)$ for $u > 0$.

The uniqueness of radial positive solutions is studied. Hence the boundary value problem

$$u'' + \frac{N-1}{r}u' + f(u) = 0, \quad r \in (a, b), \qquad u(a) = u(b) = 0$$

is considered. The uniqueness results of positive solutions are shown.

PERIODIC SOLUTIONS OF THE LORENTZ FORCE EQUATION

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Joint work with **David Arcoya and Cristian Bereanu**.

We consider the existence of T-periodic solutions of the Lorentz force equation

$$\left(\frac{q'}{\sqrt{1 - |q'|^2}} \right)' = E(t, q) + q' \times B(t, q)$$

where the electric and magnetic fields E, B are written in terms of the electric and magnetic potentials $V : [0, T] \times \mathbb{R}^3 \to \mathbb{R}$ and $W : [0, T] \times \mathbb{R}^3 \to \mathbb{R}^3$ as

$$E = -\nabla_q V - \frac{\partial W}{\partial t}, \qquad B = \operatorname{curl}_q W.$$

Following for instance [3, 4], this is the relativistic equation of motion for a single charge in the fields generated by V and W. The above equation is formally the Euler - Lagrange equation of the relativistic Lagrangian

$$\mathcal{L}(t, q, q') = 1 - \sqrt{1 - |q'|^2} + q' \cdot W(t, q) + V(t, q).$$

and also it is the Hamilton - Jacobi equation of the relativistic Hamiltonian

$$\mathcal{H}(t, p, q) = \sqrt{1 + |p - W(t, q)|^2} - 1 + V(t, q).$$

The purpose of the talk is to review some methods recently developed for the existence and multiplicity of T-periodic solutions, by using a topological degree approach [1] or a novel critical point theory [2].

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DULAC TIME FOR FAMILIES OF HYPERBOLIC SADDLE SINGULARITIES

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Joint work with **David Marín**.

We consider a smooth unfolding of a saddle point and we fix two transverse sections, the first one at the stable separatrix and the second one at the unstable separatrix. The *Dulac time* is the time that spends the flow for the transition from the first to the second transverse section. We present a structure theorem for the asymptotic expansion of the Dulac time, with the principal part expressed in terms of Roussarie's monomial scale, and the remainder having flat properties that are well preserved through the division-derivation algorithm. We also provide explicit formulae for the coefficients of the first monomials in the principal part by means of a new integral operator that generalizes Mellin transform. We explain its applicability in the study of the bifurcation diagram of the period function of the quadratic reversible centers.

SECOND-ORDER INDEFINITE SINGULAR EQUATIONS. THE PERIODIC CASE

Manuel Zamora
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In this talk we will discuss the existence of a $T-$periodic solution to the second order differential equation

$$u'' = \frac{h(t)}{u^{\lambda}},$$

where $\lambda > 0$ and the weight $h \in L(\mathbb{R}/T\mathbb{Z})$ is a sign-changing function. When $\lambda \geq 1$, the obtained results have the form of relation between the multiplicities of the zeroes of the weight function h and the order of the singularity of the nonlinear term. Nevertheless, when $\lambda \in (0, 1)$, the key ingredient to solve the aforementioned problem is connected more with the oscillation and the symmetry aspects of the weight function h than with the multiplicity of its zeroes.

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INVARIANTS OF GROUP REPRESENTATIONS, DIMENSION/DEGREE DUALITY AND NORMAL FORMS OF VECTOR FIELDS

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Joint work with **Ewa Stróżyna**.

We develop a constructive approach to the problem of polynomial first integrals for linear vector fields. As an application we obtain a new proof of the theorem of Wietzenböck about finiteness of the number of generators of the ring of constants of a linear derivation in the polynomial ring. Moreover, we propose an alternative approach to the analyticity property of the normal form reduction of a germ of vector field with nilpotent linear part in a case considered by Stolovich and Verstringe.

DYNAMICS OF DDE SYSTEM GENERALIZED FROM NICHOLSON'S EQUATION TO A TWO-PATCH ENVIRONMENT WITH DENSITY-DEPENDENT DISPERSALS

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Joint work with **Chang-Yuan Cheng and Shyan-Shiou Chen**.

We derive a model system that describes the dynamics of a single species over two patches with local dynamics governed by Nicholson's DDE and coupled by density dependent dispersals. Under a biologically meaningful assumption that the dispersal rate during the immature period depends only on the mature population, we are able to analyze model to some extent: well-posedness is confirmed, criteria for existence of a positive equilibrium are obtained, threshold

for extinction/persistence is established. We also identify a positive invariant set and establish global convergence of solutions under certain conditions. We find that although the levels of the density-dependent dispersals play no role in determining extinction/persistence, our numerical results show that they can affect, when the population is persistent, the long term dynamics including the temporal-spatial patterns and the final population sizes.

ON THE DYNAMICS OF THE ZEROS OF SOLUTIONS OF AIRY EQUATION (AND BEYOND)

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We discuss the dynamics of the zeros of entire functions in the complex plane. In particular we present the dependence of the zeros of solutions of the Airy equation on two parameters introduced in the equation. The parameters characterize the general solution of the equation. A system of infinitely many nonlinear evolution differential equations are obtained, displaying interesting properties. The possibility to extend the approach to other entire functions in the complex plane will be discussed.