DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D32

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D3

KAM THEORY FOR SECONDARY TORI

Luigi Chierchia
Roma Tre University, Italy
Joint work with **Luca Biasco**.

As well known, classical KAM (Kolmogorov, Arnold, Moser) theory deals with the persistence, under small perturbations, of real-analytic (or smooth) Lagrangian tori for nearly-integrable non-degenerate Hamiltonian systems. In this talk I will present a new *uniform* KAM theory apt to deal also with secondary tori, i.e., maximal invariant tori (with different homotopy) "generated" by the perturbation (and that do not exist in the integrable limit). The word "uniform" means that primary and secondary tori are constructed simultaneously; in particular, in the case of Newtonian mechanical systems on $\mathbf{T}^d$, it is proven that, for generic perturbations, the union of primary and secondary tori leave out a region of order $\varepsilon|\log\varepsilon|^a$, if ε is the norm of the perturbation, in agreement (up to the logarithmic correction) with a conjecture by Arnold, Kozlov and Neishtadt.

Some of these results have been announced in the note [1].

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SOME GEOMETRIC MECHANISMS FOR ARNOLD DIFFUSION

Rafael de la Llave
Georgia Institute of Technology, USA

We consider the problem whether small perturbations of integrable mechanical systems can have very large effects. Since the work of Arnold in 1964, it is known that there are situations where the perturbations can accumulate. This can be understood by noting that the small perturbations generate some invariant structures that, with their stable and unstable manifolds can cover a large region in phase space. We will present recent developments in identifying these invariant objects, both in finite and in infinite dimensions.

DIFFERENTIAL EQUATIONS FOR NETWORK CENTRALITY

Desmond Higham
University of Edinburgh, UK

I will derive and discuss two circumstances where ODEs arise in the study of large, complex networks. In both cases, the overall aim is to identify the most important nodes in a network- this task is useful, for example, in digital marketing, security and epidemiology. In one case, we define our node centrality measure using the concept of nonbacktracking walks. This requires us to derive an expression for an exponential-type generating function associated with the walk counts of different length. Solving the ODE leads to a computationally useful characterisation of the centrality measure. In another case, we are presented with a time-ordered sequence of networks; for example, recording who emailed who over each one-minute time-window. Here, by considering the asymptotic limit as the window size tends to zero, we arrive at a limiting ODE that may be treated with a numerical method. Results for both algorithms will be illustrated on real network examples.

ROBUST CHAOS: A TALE OF BLENDERS, THEIR COMPUTATION, AND THEIR DESTRUCTION

Hinke Osinga
University of Auckland, New Zealand
Joint work with **Stephanie Hittmeyer, Bernd Krauskopf, and Katsutoshi Shinohara**.

A blender is an intricate geometric structure of a three- or higher-dimensional diffeomorphism [1]. Its characterising feature is that its invariant manifolds behave as geometric objects of a dimension that is larger than expected from the dimensions of the manifolds themselves. We introduce a family of three-dimensional Hénon-like maps and study how it gives rise to an explicit example of a blender [2, 3]. We employ our advanced numerical techniques to present images of blenders and their associated one-dimensional stable manifolds. Moreover, we develop an effective and accurate numerical test to verify what we call the *carpet property* of a blender. This approach provides strong numerical evidence for the existence of the blender over a large parameter range, as well as its disappearance and geometric properties beyond this range. We conclude with a discussion of the relevance of the carpet property for chaotic attractors.

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THE JOINT SPECTRAL RADIUS AND FUNCTIONAL EQUATIONS: A RECENT PROGRESS

Vladimir Protasov
University of L'Aquila, Italy & Lomonosov Moscow State University, Russia

Joint spectral radius of matrices have been used since late eighties as a measure of stability of linear switching dynamical systems. Nearly in the same time it has found important applications in the theory of refinement equations (linear difference equations with a contraction of the argument), which is a key tool in the construction of compactly supported wavelets and of subdivision schemes in approximation theory and design of curves and surfaces. However, the computation or even estimation of the joint spectral radius is a hard problem. It was shown by Blondel and Tsitsiklis that this problem is in general algorithmically undecidable. Nevertheless recent geometrical methods [1,2,3,4] make it possible to efficiently estimate this value or even find it precisely for the vast majority of matrices. We discuss this issue and formulate some open problems.

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OPTIMAL CONTROL AND APPLICATIONS TO AEROSPACE

Emmanuel Trélat
Sorbonne Université, France

I will report on nonlinear optimal control theory and show how it can be used to address problems in aerospace, such as orbit transfer. The knowledge resulting from the Pontryagin maximum principle is in general insufficient for solving adequately the problem, in particular due to the difficulty of initializing the shooting method. I will show how the shooting method can be successfully combined with numerical homotopies, which consist of deforming continuously a problem towards a simpler one. In view of designing low-cost interplanetary space missions, optimal control can also be combined with dynamical system theory, using the nice dynamical properties around Lagrange points that are of great interest for mission design.

SMALL DIVISORS AND NORMAL FORMS

Warwick Tucker
Uppsala University, Sweden
Joint work with **Zbigniew Galias**.

In this talk, we will discuss the computational challenges of computing trajectories of a non-linear ODE in a region close to a saddle-type fixed-point. By introducing a carefully selected close to identity change of variables, we can bring the non-linear ODE into an "almost" linear system. This normal form system has an explicit transfer-map that transports trajectories away from the fixed point in a controlled manner. Determining the domain of existence for such a change of variables poses some interesting computational challenges. The proposed method is quite general, and can be extended to the complex setting with spiral saddles. It is also completely constructive which makes it suitable for practical use. We illustrate the use of the method by a few examples.

ORTHOGONAL POLYNOMIALS AND PAINLEVÉ EQUATIONS

Walter Van Assche
KU Leuven, Belgium

Painlevé equations are nonlinear differential equations for which the branch points do not depend on the initialcondition (no movable branch points). There are also discrete Painlevé equations which are non-linearrecurrence relations with enough structure (symmetry and geometry) that make them integrable.Both the discrete and continuous Painlevé equations appear in a natural way in the theory of orthogonalpolynomials. The recurrence coefficients of certain families of orthogonal polynomials often satisfya discrete Painlevé equation. The Toda equations describing the movement of particles with an exponentialinteraction with their neighbors, is equivalent to an exponential modification $e^{xt}\,d\mu(x)$ of the orthogonality measure $d\mu$ for a family of orthogonal polynomials, and the corresponding recurrencecoefficients satisfy the Toda equations, which is a system of differential-difference equations. Combining this with the discrete Painlevé equations then gives a Painlevé differential equation.We will illustrate this by a number of examples. The relevant solutions of these Painlevé equationsare usually in terms of known special functions, such as the Bessel functions, the Airy function, parabolic cylinder functions, or (confluent) hypergeometric functions.

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DELAY EQUATIONS AND TWIN SEMIGROUPS

Sjoerd Verduyn Lunel
Utrecht University, Netherlands
Joint work with **Odo Diekmann**.

A delay equation is a rule for extending a function of time towards the future on the basis of the (assumed to be) known past. By translation along the extended function (i.e., by updating the history), one defines a dynamical system. If one chooses as state-space the continuous initial functions, the translation semigroup is continuous, but the initial data corresponding to the fundamental solution is not contained in the state space.

In ongoing joint work with Odo Diekmann, we choose as state space the space of bounded Borel functions and thus sacrifice strong continuity in order to gain a simple description of the variation-of-constants formula.

The aim of the lecture is to introduce the perturbation theory framework of twin semigroups on a norming dual pair of spaces, to show how renewal equations fit in this framework and to sketch how neutral equations can be covered. The growth of an age-structured population serves as a pedagogical example.

DYNAMICAL SYSTEM APPROACH TO SPECTRAL THEORY OF QUASI-PERIODIC SCHRÖDINGER OPERATORS

Jiangong You
Nankai University, China

The spectral theory of quasiperiodic operators is a fascinating field which continuously attracts a lot of attentions for its rich background in quantum physics as well as its rich connections with many mathematical theories and methods. In this talk, I will briefly introduce the problems in this field and their connections with dynamical system. I will also talk about some recent results joint with Avila, Ge, Leguil, Zhao and Zhou on both spectrum and spectral measure by reducibility theory in dynamical systems.

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TALKS OF SESSION D32

BIFURCATIONS IN PERIODIC IDEs

Christian Aarset
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Joint work with **Christian Pötzsche**.

In theoretical ecology, one often models population growth with the help of discrete-time difference equations. One method to account for the effects of dispersal throughout the habitat is to employ *integrodifference equations* [1], or IDEs, as opposed to employing scalar difference equations. Given a compact habitat $\Omega \subset \mathbb{R}^d$, usually with $d = 1, 2, 3$, together with some appropriate parameter space Λ, we consider IDEs on the form

$$u_{t+1} := \int_{\Omega} f(\cdot, y, u_t(y), \alpha)dy \tag{1}$$

with $u_t \in C(\Omega)$ for all $t \in \mathbb{N}$, where $f : \Omega \times \Omega \times \mathbb{R} \times \Lambda \to \mathbb{R}$ is some appropriate function; a commonly employed form for such f is e.g. $f(x, y, z, \alpha) := k(x, y)g(y, z, \alpha)$, where k is some dispersal kernel (e.g. Laplace, Gaussian) and g is some parameter-dependent growth function (e.g. Beverton-Holt, Ricker).

One is frequently interested in the stability behaviour of *fixed points*, solutions u^* of (1). However, certain IDEs, in particular - but not limited to - those using the Ricker growth function, may feature transfer of stability from a branch of fixed points to a branch of two- or higher-*periodic solutions*, solutions of the *iterated equation*. We explore such *flip bifurcations* in details, and generalize this theory to cover bifurcations of *periodic solutions* of any integer period, with the particular goal of formulating our assumptions so that they can easily be verified numerically.

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LYAPUNOV SPECTRUM ASSIGNABILITY PROBLEM OF DYNAMICAL SYSTEMS

Artur Babiarz
Silesian University of Technology, Poland
Joint work with **Irina Banshchikova, Adam Czornik, Evgenii Makarov, Michał Niezabitowski, and Svetlana Popova**.

For discrete linear time-varying systems with bounded coefficients, the pole assignment problem utilizing linear state feedback is discussed. It is shown that uniform complete controllability is sufficient for the Lyapunov exponents being arbitrarily assignable by choosing a suitable feedback. Our aim is to prove that all the systems from the closure (in the topology of pointwise convergence) of all shifts of the original system have assignable Lyapunov spectrum if and only if the original system is uniformly completely controllable. Using an appropriate time-varying linear feedback we obtain sufficient conditions to place the Lyapunov spectrum of the closed-loop system in an arbitrary position within some neighborhood of the Lyapunov spectrum of the free system. Moreover, we prove that diagonalizability, Lyapunov regularity and stability of the Lyapunov spectrum each separately are the required sufficient conditions provided that the open-loop system is uniformly completely controllable.

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CONCAVE AND CONCAVE CARRYING SIMPLICES

Stephen Baigent
University College London, UK

The carrying simplex is a codimension-one invariant hypersurface that is the common boundary of the basins of repulsion of the origin and infinity of both continuous- and discrete-time competitive Kolmogorov systems [1, 2].

Geometrically carrying simplices are 'nice' invariant manifolds. They project radially one-to-one and onto the unit probability simplex and are graphs of locally Lipschitz functions. Moreover, in some cases they may be graphs of convex, concave or saddle-like functions [4, 3, 5, 6].

I will introduce the carrying simplex and discuss how the bending of hyperplanes under the map can be used to determine when the carrying simplex is convex or concave.

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ON THE SOLUTIONS OF RICCATI DIFFERENCE EQUATION VIA FIBONACCI NUMBERS

Inese Bula and Diāna Mežecka
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A difference equation of the form

$$x_{n+1} = \frac{\alpha + \beta x_n}{A + B x_n}, \quad n = 0, 1, ...,$$

where the parameters α, β, A, B and the initial condition x_0 are real numbers is called a Riccati difference equation. This equation has been studied in many articles (for example, see general review in [1]). In [2, 3] authors studied special cases of Riccati difference equation whose solutions can be expressed via Fibonacci numbers.

In our talk we consider a Riccati difference equation in the form

$$x_{n+1} = \frac{F_m + F_{m-1} x_n}{F_{m+1} + F_m x_n}, \quad n = 0, 1, ..., \tag{1}$$

where $F_0 = 0$, $F_1 = 1$, ..., $F_{m+1} = F_m + F_{m-1}$, $m = 1, 2, ...$, are Fibonacci numbers. We show some properties of equations (1), including following result.

Theorem. For every $m = 1, 2, ...$, and every initial condition $x_0 \neq -\frac{F_{k+1}}{F_k}$, $k = 1, 2, ...$, the solution of equation (1) is in the form

$$x_n = \frac{F_{mn} + F_{mn-1} x_0}{F_{mn+1} + F_{mn} x_0}, \quad n = 1, 2, \dots .$$

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DICHOTOMY SPECTRUM AND ALMOST TOPOLOGICAL CONJUGACY ON NONAUTONOMOUS UNBOUNDED DIFFERENCE SYSTEM

Álvaro Castañeda
University of Chile, Chile
Joint work with **Gonzalo Robledo**.

We will consider the nonautonomous linear system

$$x(n+1) = A(n)x(n) \tag{1}$$

where $x(n)$ is a column vector of $\mathbb{R}^d$ and the matrix function $n \mapsto A(n) \in \mathbb{R}^{d\times d}$ is non singular. We also assume that (1) has an exponential dichotomy on $\mathbb{Z}$ with projector $P = I$ (see [1] for a formal definition). We also consider the perturbed system

$$w(n+1) = A(n)w(n) + f(n, w(n)) \tag{2}$$

where $f : \mathbb{Z} \times \mathbb{R}^d \to \mathbb{R}^d$ is continuous in $\mathbb{R}^d$ is a Lipschitz function such that $n \mapsto f(n, 0)$ is bounded for any $n \in \mathbb{Z}$. We will present a result with sufficient conditions ensuring that (1) and (2) are almost topologically equivalent, namely the existence of a map $H : \mathbb{Z} \times \mathbb{R}^d \to \mathbb{R}^d$ with the following properties: i) For each fixed $n \in \mathbb{Z}$, the map $u \mapsto H(n, u)$ is a bijection. ii) For any fixed $n \in \mathbb{Z}$, the maps $u \mapsto H(n, u)$ and $u \mapsto H^{-1}(n, u) = G(n, u)$ are continuous with the possible exception of a set with Lebesgue measure zero. iii) If $x(n)$ is a solution of (1), then $H[n, x(n)]$ is a solution of (2). Similarly, if $w(n)$ is a solution of (2), then $G[n, w(n)]$ is a solution of (1). This result can also be seen as a generalization of a continuous result obtained by F. Lin in [2].

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DISCRETE TIME-VARYING FRACTIONAL LINEAR EQUATIONS AS VOLTERRA CONVOLUTION EQUATIONS

Adam Czornik

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Joint work with **Pham The Anh, Artur Babiarz, Michał Niezabitowski, and Stefan Siegmund**.

We study the discrete-time fractional linear systems. We show how the different type (Caputo, Riemann-Liouville, forward and backward) of fractional linear difference equation may be converted to Volterra convolution equation. Using this representation we obtain some results about rate of convergency and divergency of solutions and variation of constant formulae. Moreover we show that the norm of difference between two different solution can not tends to infinity faster than a polynomial which degree depends of the fractional order of difference.

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DISCRETE BOUNDARY VALUE PROBLEMS ON UNBOUNDED DOMAINS

Zuzana Došlá

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Joint work with **Mauro Marini and Serena Matucci**.

We study the boundary value problem

$$\begin{cases} \Delta(a_n\Phi(\Delta x_n)) = b_n F(x_{n+1}), & n \in \mathbb{N} \\ x_1 = c, \quad x_n > 0, \quad \lim_{n\to\infty} x_n = d, \end{cases} \tag{1}$$

where Δ is the forward difference operator $\Delta x_n = x_{n+1} - x_n$, Φ is an increasing odd homeomorphisms, $\Phi : (-\rho, \rho) \to (-\sigma, \sigma)$ such that $\Phi(u)u > 0$ for $u \neq 0$, and $\rho, \sigma \leq \infty$. We assume that the sequences (a_n), (b_n) are positive, and boundary conditions satisfy $c > 0$ and $d \geq 0$. Solutions of (1) with the terminal condition $\lim_{n\to\infty} x_n = 0$ are usually called decaying solution.

Problem (1) appears in the discretization process for searching spherically symmetric solutions of certain nonlinear elliptic differential equations with generalized phi-Laplacian. The case of noncompact domains seems to be of particular interest in view of applications to radially symmetric solutions to PDEs on the exterior of a ball.

Prototypes of Φ are the classical Φ-Laplacian,

$$\Phi_p(u) = |u|^{p-2}u, \quad p \geq 1\,;$$

and when $\sigma < \infty$ and $\rho < \infty$ operators

$$\Phi_C(u) = \frac{u}{\sqrt{1+|u|^2}} \quad \text{and} \quad \Phi_R(u) = \frac{u}{\sqrt{1-|u|^2}}$$

arising in studying radial symmetric solutions of partial differential equations with the mean curvature and the relativity operator, respectively.

If Φ is the classical Φ-Laplacian, the solvability of (1) has been investigated in [2], using properties of the recessive solution to suitable half-linear difference equations, a half-linearization technique and a fixed point theorem in Frechét spaces (see also [3]). Problem (1) is also motivated by [1] where general Φ has been considered and extremal solutions have been investigated in case that (b_n) is negative.

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DELAY DIFFERENCE EQUATIONS: PERMANENCE AND THE STRUCTURE OF THE GLOBAL ATTRACTOR

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Alpen-Adria Universität Klagenfurt, Austria
Joint work with **Christian Pötzsche**.

In the first part of the talk we give sufficient conditions on the uniform boundedness and permanence of non-autonomous multiple delay difference equations of the form

$$x_{k+1} = x_k f_k(x_{k-d}, \ldots, x_{k-1}, x_k),$$

where $f_k\colon D \subseteq (0,\infty)^{d+1} \to (0,\infty)$. This also implies the existence of the global (pullback) attractor, provided the right-hand side is continuous.

In the second part, under some feedback conditions the right-hand side, we give a so-called Morse decomposition of the global attractor for equations of the form $x_{k+1} = g(x_{k-d}, x_k)$. The decomposition is based on an integer valued Lyapunov functional introduced by J. Mallet-Paret and G. Sell.

Both results are applicable for a wide range of single species discrete time population dynamical models, such as models by Ricker, Pielou, Mackey-Glass, Wazewska-Lasota, and Clark.

ON CONSENSUS UNDER DoS ATTACK IN THE MULTIAGENT SYSTEMS

Ewa Girejko
Bialystok University of Technology, Poland
Joint work with **Agnieszka B. Malinowska**.

In the paper multiagent systems under Denial-of-Service (DoS) attack are considered. We provide convergence results to ensure the consensus in the system under the attack. Since DoS attack is usually unpredictable with respect to duration of time and lasts one second or more, we examine the problem on various time domains.

OPTIMAL CONTROL OF FRACTIONAL MULTI-AGENT SYSTEMS

Agnieszka Malinowska
Bialystok University of Technology, Poland
Joint work with **Tatiana Odzijewicz**.

We deal with control strategies for discrete-time fractional multi-agent systems. By using the discrete fractional order operator we introduce memory effects to the considered problem. Necessary optimality conditions for discrete-time fractional optimal control problems with single- and double-summator dynamics are proved. We demonstrate the validity of the proposed control strategy by numerical examples.

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ASYMPTOTIC PROBLEMS FOR SECOND ORDER NONLINEAR DIFFERENCE EQUATIONS WITH DEVIATING ARGUMENT

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Joint work with **Zuzana Došlá** .

A fixed point approach, based on Schauder linearization device in the Frechét space of all the sequences, is presented and compared with corresponding results in the space of continuous functions, extending results in [3]. As an applications, the problem of the existence of the so-called intermediate solutions is analyzed for the half-linear and sublinear Emden-Fowler type equations with deviating argument

$$\Delta(a_n|\Delta x_n|^{\alpha} \operatorname{sgn} \Delta x_n) + b_n|x_{n+q}|^{\beta} \operatorname{sgn} x_{n+q} = 0, \tag{1}$$

where Δ is the forward difference operator, $a = \{a_n\}$, $b = \{b_n\}$ are positive real sequences, $0 < \beta \leq \alpha$ and $q \in \mathbb{Z}$. In particular, we analyze the effect of the deviating argument on the existence of unbounded nonoscillatory solutions for (1), by means of a comparison with the equation

$$\Delta(a_n|\Delta y_n|^{\alpha} \operatorname{sgn} \Delta y_n) + b_n|y_{n+1}|^{\beta} \operatorname{sgn} y_{n+1} = 0. \tag{2}$$

As a consequence, necessary and sufficient conditions for the existence of intermediate solutions for (1) (that is, eventually positive solutions x s.t. $\lim_n x_n = +\infty$, $\lim_n a_n|\Delta x_n|^{\alpha} = 0$) are given. The results presented generalize some in [1] in case $\alpha = \beta$, and in [2] when $\alpha > \beta$.

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TBA

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ASYMPTOTIC PROPERTIES OF SOLUTIONS TO SUM-DIFFERENCE EQUATIONS OF VOLTERRA TYPE

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Volterra difference equations appeared as a discretization of Volterra integral and integro-differential equations. They also often arise during the mathematical modeling of some real life situations where the current state is determined by the whole previous history. In this talk we consider some difference equations of Volterra type. In particular we discuss the equations of the form

$$\Delta(r_n \Delta x_n) = b_n + \sum_{k=1}^{n} K(n,k) f(x_k).$$

We give sufficient conditions for the existence of a solution x of the above equation with the property

$$x_n = y_n + \mathrm{o}(n^s),$$

where y is a given solution of the equation $\Delta(r_n \Delta y_n) = b_n$ and $s \in (-\infty, 0]$. We show also applications of the obtained results to a linear Volterra equation. Sufficient conditions for the existence of asymptotically periodic solutions will be discussed as well.

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OPTIMAL LEADER-FOLLOWER CONTROL FOR THE FRACTIONAL OPINION FORMATION MODEL

Tatiana Odzijewicz
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Joint work with **Ricardo Almeida and Agnieszka B. Malinowska**.

This work deals with an opinion formation model, that obeys a nonlinear system of fractional-order differential equations. We introduce a virtual leader in order to attain a consensus.

Sufficient conditions are established to ensure that the opinions of all agents globally asymptotically approach the opinion of the leader. We also address the problem of designing optimal control strategies for the leader so that the followers tend to consensus in the most efficient way. A variational integrator scheme is applied to solve the leader-follower optimal control problem. Finally, in order to verify the theoretical analysis, several particular examples are presented.

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EXPONENTIAL DICHOTOMY AND SEPARATION IN LINEAR DIFFERENCE EQUATIONS

Kenneth James Palmer
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Joint work with **Flaviano Battelli**.

We consider linear difference equations $x(n{+}1) = A(n)x(n)$, in which $A(n)$ may not be invertible or bounded. The main issues considered here are robustness (or roughness) and the relation between a triangular system and its corresponding diagonal system. In general, exponential separation is weaker than exponential dichotomy but, for certain systems, it turns out that in some sense exponential separation implies exponential dichotomy. Differences between the differential equations case and the difference equations case are highlighted.

ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS OF LINEAR DIFFERENCE EQUATIONS

Mihály Pituk
University of Pannonia, Hungary

In this talk, we will summarize some results on the asymptotic behavior of the positive solutions of linear difference equations. Under appropriate assumptions, we will study the growth rates and the existence of wieghted limits of the positive solutions.

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GLOBAL ATTRACTIVITY AND DISCRETIZATION IN INTEGRODIFFERENCE EQUATIONS

Christian Pötzsche
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Integrodifference equations are popular models in theoretical ecology to describe the temporal evolution and spatial dispersal of populations having nonoverlapping generations. As a contribution to the numerical dynamics of such infinite-dimensional dynamical systems, we establish

that global attractivity of periodic solutions is robust under a wide class of spatial discretizations. Beyond robustness also the convergence order of the numerical schemes is preserved.

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REFINED DISCRETE REGULAR VARIATION AND ITS APPLICATIONS IN DIFFERENCE EQUATIONS

Pavel Řehák

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We introduce a new class of the so-called regularly varying sequences with respect to an auxiliary sequence τ, and state its properties. This class, on one hand, generalizes regularly varying sequences. On the other hand, it refines them and makes it possible to do a more sophisticated analysis in applications. We show a close connection with regular variation on time scales; thanks to this relation, we can use the existing theory on time scales to develop discrete regular variation with respect to τ. We reveal also a connection with generalized regularly varying functions. As an application, we study asymptotic behavior of solutions to linear difference equations; we obtain generalization and extension of known results. The theory also yields, as a by-product, a knew view on the Kummer type test for convergence of series, which generalizes, among others, Raabe's test and Bertrand's test.

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CONSENSUS OF MULTI-AGENTS SYSTEMS ON ARBITRARY TIME SCALE

Ewa Schmeidel
University of Bialystok, Poland
Joint work with **Urszula Ostaszewska and Małgorzata Zdanowicz**.

In my talk an emergence of leader-following model based on graph theory on the arbitrary time scales is investigated. It means that the step size is not necessarily constant but it is a function of time. We propose and prove conditions ensuring a leader-following consensus for any time scales using Grönwall inequality. The presented results are illustrated by examples.

References

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SINGULAR STURMIAN THEORY FOR WEAKLY DISCONJUGATE LINEAR HAMILTONIAN DIFFERENTIAL SYSTEMS

Peter Šepitka
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Joint work with **Roman Šimon Hilscher**.

In this talk we introduce several new results in the Sturmian theory of weakly disconjugate (or equivalently, eventually controllable) linear Hamiltonian systems. We present singular comparison theorems on unbounded intervals for two nonoscillatory systems satisfying the Sturmian majorant condition and the Legendre condition. In particular, we show exact formulas and optimal estimates for the numbers of proper focal points of conjoined bases of these systems. This topic was infrequently studied in the literature and the validity of singular comparison theorems on unbounded intervals for general uncontrollable setting is an open problem so far. The presented results complete and generalize the previously obtained (i) singular Sturmian comparison/separation theorems on unbounded intervals by O. Došlý and W. Kratz in [1], and by the author jointly with R. Šimon Hilscher in [3], (ii) as well as the Sturmian comparison theorems on compact intervals by R. Šimon Hilscher in [4] and by J. Elyseeva in [2].

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A HILBERT SPACE APPROACH TO FRACTIONAL DIFFERENCE EQUATIONS

Stefan Siegmund
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Joint work with **Pham The Anh, Artur Babiarz, Adam Czornik, Konrad Kitzing, Michał Niezabitowski, Sascha Trostorff, and Hoang The Tuan**.

We formulate fractional difference equations of Riemann-Liouville and Caputo type in a functional analytical framework. Main results are existence of solutions on Hilbert space-valued weighted sequence spaces and a condition for stability of linear fractional difference equations. Using a functional calculus, we relate the fractional sum to fractional powers of the operator $1-\tau^{-1}$ with the right shift τ^{-1} on weighted sequence spaces. Causality of the solution operator plays a crucial role for the description of initial value problems

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THE STORY OF FOCAL POINT IN DISCRETE STURMIAN THEORY

Roman Šimon Hilscher
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Joint work with **Peter Šepitka**.

We will discuss the development of the concepts of generalized zeros and focal points for second order difference equations and symplectic difference systems in the relation with the validity of the Sturmian separation and comparison theorems. Our aim is to present recent progress in this

area by discussing singular Sturmian theory for possibly uncontrollable symplectic difference systems on unbounded intervals. We will also present a simple application of the new concept in disconjugacy criteria for the second order Sturm-Liouville difference equations on unbounded intervals.

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A CONTINUATION PRINCIPLE FOR FREDHOLM MAPS AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

Robert Skiba
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Joint work with **Christian Pötzsche**.

In this talk we are going to present an abstract and flexible continuation theorem for zeros of parametrized Fredholm maps between Banach spaces. It guarantees not only the existence of zeros to corresponding equations but also that they form a continuum for parameters from a connected manifold. Our basic tools will be transfer maps and a specific topological degree. Next, we will explain how using an abstract and flexible continuation theorem to find global branches of homoclinic solutions for parametrized nonautonomous ordinary differential equations. Our approach will be based on degree-theoretical arguments. In particular, Landesman-Lazer conditions will be proposed to obtain the existence of homoclinic solutions by means of a nonzero degree.

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INTEGRABILITY OF DIFFERENCE EQUATIONS WITH BINOMIAL SERIES

Zhi-Tao Wen
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Joint work with **Katsuya Ishizaki**.

We consider binomial series $\sum_{n=0}^{\infty} a_n z^{\underline{n}}$, where $z^{\underline{n}} = z(z-1)\cdots(z-n+1)$. Integrability by binomial series is concerned for difference equations. In this talk, we consider a formal solution of a difference equation written by binomial series. Further, we discuss conditions of convergence of these formal solutions to find a sufficient condition for meromorphic solutions, and investigate the order of growth of them. As an application, we construct a difference Riccati equation possessing a transcendental meromorphic solution of order 1/2.