DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D33

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GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D3

KAM THEORY FOR SECONDARY TORI

Luigi Chierchia
Roma Tre University, Italy
Joint work with **Luca Biasco**.

As well known, classical KAM (Kolmogorov, Arnold, Moser) theory deals with the persistence, under small perturbations, of real-analytic (or smooth) Lagrangian tori for nearly-integrable non-degenerate Hamiltonian systems. In this talk I will present a new *uniform* KAM theory apt to deal also with secondary tori, i.e., maximal invariant tori (with different homotopy) "generated" by the perturbation (and that do not exist in the integrable limit). The word "uniform" means that primary and secondary tori are constructed simultaneously; in particular, in the case of Newtonian mechanical systems on $\mathbf{T}^d$, it is proven that, for generic perturbations, the union of primary and secondary tori leave out a region of order $\varepsilon|\log\varepsilon|^a$, if ε is the norm of the perturbation, in agreement (up to the logarithmic correction) with a conjecture by Arnold, Kozlov and Neishtadt.

Some of these results have been announced in the note [1].

References

[1] L. Biasco, L. Chierchia, *On the measure of Lagrangian invariant tori in nearly-integrable mechanical systems*, Rend. Lincei Mat. Appl. **26** (2015), 423-432.

SOME GEOMETRIC MECHANISMS FOR ARNOLD DIFFUSION

Rafael de la Llave
Georgia Institute of Technology, USA

We consider the problem whether small perturbations of integrable mechanical systems can have very large effects. Since the work of Arnold in 1964, it is known that there are situations where the perturbations can accumulate. This can be understood by noting that the small perturbations generate some invariant structures that, with their stable and unstable manifolds can cover a large region in phase space. We will present recent developments in identifying these invariant objects, both in finite and in infinite dimensions.

DIFFERENTIAL EQUATIONS FOR NETWORK CENTRALITY

Desmond Higham
University of Edinburgh, UK

I will derive and discuss two circumstances where ODEs arise in the study of large, complex networks. In both cases, the overall aim is to identify the most important nodes in a network-this task is useful, for example, in digital marketing, security and epidemiology. In one case, we define our node centrality measure using the concept of nonbacktracking walks. This requires us to derive an expression for an exponential-type generating function associated with the walk counts of different length. Solving the ODE leads to a computationally useful characterisation of the centrality measure. In another case, we are presented with a time-ordered sequence of networks; for example, recording who emailed who over each one-minute time-window. Here, by considering the asymptotic limit as the window size tends to zero, we arrive at a limiting ODE that may be treated with a numerical method. Results for both algorithms will be illustrated on real network examples.

ROBUST CHAOS: A TALE OF BLENDERS, THEIR COMPUTATION, AND THEIR DESTRUCTION

Hinke Osinga
University of Auckland, New Zealand
Joint work with **Stephanie Hittmeyer, Bernd Krauskopf, and Katsutoshi Shinohara**.

A blender is an intricate geometric structure of a three- or higher-dimensional diffeomorphism [1]. Its characterising feature is that its invariant manifolds behave as geometric objects of a dimension that is larger than expected from the dimensions of the manifolds themselves. We introduce a family of three-dimensional Hénon-like maps and study how it gives rise to an explicit example of a blender [2, 3]. We employ our advanced numerical techniques to present images of blenders and their associated one-dimensional stable manifolds. Moreover, we develop an effective and accurate numerical test to verify what we call the *carpet property* of a blender. This approach provides strong numerical evidence for the existence of the blender over a large parameter range, as well as its disappearance and geometric properties beyond this range. We conclude with a discussion of the relevance of the carpet property for chaotic attractors.

References

[1] C. Bonatti, S. Crovisier, L.J. Díaz, A. Wilkinson, *What is... a blender?*, Not. Am. Math. Soc. **63** (2016), 1175-1178.

[2] L.J. Díaz, S. Kiriki, K. Shinohara, *Blenders in centre unstable Hénon-like families: with an application to heterodimensional bifurcations*, Nonlinearity **27** (2014), 353-378.

[3] S. Hittmeyer, B. Krauskopf, H.M. Osinga, K. Shinohara, *Existence of blenders in a Hénon-like family: geometric insights from invariant manifold computations*, Nonlinearity **31** (2018), R239-R267.

THE JOINT SPECTRAL RADIUS AND FUNCTIONAL EQUATIONS: A RECENT PROGRESS

Vladimir Protasov
University of L'Aquila, Italy & Lomonosov Moscow State University, Russia

Joint spectral radius of matrices have been used since late eighties as a measure of stability of linear switching dynamical systems. Nearly in the same time it has found important applications in the theory of refinement equations (linear difference equations with a contraction of the argument), which is a key tool in the construction of compactly supported wavelets and of subdivision schemes in approximation theory and design of curves and surfaces. However, the computation or even estimation of the joint spectral radius is a hard problem. It was shown by Blondel and Tsitsiklis that this problem is in general algorithmically undecidable. Nevertheless recent geometrical methods [1,2,3,4] make it possible to efficiently estimate this value or even find it precisely for the vast majority of matrices. We discuss this issue and formulate some open problems.

References

[1] N. Guglielmi, V.Yu. Protasov, *Exact computation of joint spectral characteristics of matrices*, Found. Comput. Math **13** (2013), 37-97.

[2] C. Möller, U. Reif, *A tree-based approach to joint spectral radius determination*, Linear Alg. Appl. **563** (2014), 154-170.

[3] N. Guglielmi, V.Yu. Protasov, *Invariant polytopes of linear operators with applications to regularity of wavelets and of subdivisions*, SIAM J. Matrix Anal. **37** (2016), 18-52.

[4] T. Mejstrik, *Improved invariant polytope algorithm and applications*, arXiv:1812.03080.

OPTIMAL CONTROL AND APPLICATIONS TO AEROSPACE

Emmanuel Trélat
Sorbonne Université, France

I will report on nonlinear optimal control theory and show how it can be used to address problems in aerospace, such as orbit transfer. The knowledge resulting from the Pontryagin maximum principle is in general insufficient for solving adequately the problem, in particular due to the difficulty of initializing the shooting method. I will show how the shooting method can be successfully combined with numerical homotopies, which consist of deforming continuously a problem towards a simpler one. In view of designing low-cost interplanetary space missions, optimal control can also be combined with dynamical system theory, using the nice dynamical properties around Lagrange points that are of great interest for mission design.

SMALL DIVISORS AND NORMAL FORMS

Warwick Tucker
Uppsala University, Sweden
Joint work with **Zbigniew Galias**.

In this talk, we will discuss the computational challenges of computing trajectories of a non-linear ODE in a region close to a saddle-type fixed-point. By introducing a carefully selected close to identity change of variables, we can bring the non-linear ODE into an "almost" linear system. This normal form system has an explicit transfer-map that transports trajectories away from the fixed point in a controlled manner. Determining the domain of existence for such a change of variables poses some interesting computational challenges. The proposed method is quite general, and can be extended to the complex setting with spiral saddles. It is also completely constructive which makes it suitable for practical use. We illustrate the use of the method by a few examples.

ORTHOGONAL POLYNOMIALS AND PAINLEVÉ EQUATIONS

Walter Van Assche
KU Leuven, Belgium

Painlevé equations are nonlinear differential equations for which the branch points do not depend on the initialcondition (no movable branch points). There are also discrete Painlevé equations which are non-linearrecurrence relations with enough structure (symmetry and geometry) that make them integrable.Both the discrete and continuous Painlevé equations appear in a natural way in the theory of orthogonalpolynomials. The recurrence coefficients of certain families of orthogonal polynomials often satisfya discrete Painlevé equation. The Toda equations describing the movement of particles with an exponentialinteraction with their neighbors, is equivalent to an exponential modification $e^{xt}\,d\mu(x)$ of the orthogonality measure $d\mu$ for a family of orthogonal polynomials, and the corresponding recurrencecoefficients satisfy the Toda equations, which is a system of differential-difference equations. Combining this with the discrete Painlevé equations then gives a Painlevé differential equation.We will illustrate this by a number of examples. The relevant solutions of these Painlevé equationsare usually in terms of known special functions, such as the Bessel functions, the Airy function, parabolic cylinder functions, or (confluent) hypergeometric functions.

References

[1] W. Van Assche, *Orthogonal Polynomials and Painlevé Equations*, Australian Mathematical Society Lecture Notes **27**, Cambridge University Press, (2018).

DELAY EQUATIONS AND TWIN SEMIGROUPS

Sjoerd Verduyn Lunel
Utrecht University, Netherlands
Joint work with **Odo Diekmann**.

A delay equation is a rule for extending a function of time towards the future on the basis of the (assumed to be) known past. By translation along the extended function (i.e., by updating the history), one defines a dynamical system. If one chooses as state-space the continuous initial functions, the translation semigroup is continuous, but the initial data corresponding to the fundamental solution is not contained in the state space.

In ongoing joint work with Odo Diekmann, we choose as state space the space of bounded Borel functions and thus sacrifice strong continuity in order to gain a simple description of the variation-of-constants formula.

The aim of the lecture is to introduce the perturbation theory framework of twin semigroups on a norming dual pair of spaces, to show how renewal equations fit in this framework and to sketch how neutral equations can be covered. The growth of an age-structured population serves as a pedagogical example.

DYNAMICAL SYSTEM APPROACH TO SPECTRAL THEORY OF QUASI-PERIODIC SCHRÖDINGER OPERATORS

Jiangong You
Nankai University, China

The spectral theory of quasiperiodic operators is a fascinating field which continuously attracts a lot of attentions for its rich background in quantum physics as well as its rich connections with many mathematical theories and methods. In this talk, I will briefly introduce the problems in this field and their connections with dynamical system. I will also talk about some recent results joint with Avila, Ge, Leguil, Zhao and Zhou on both spectrum and spectral measure by reducibility theory in dynamical systems.

References

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[5] L. Ge, J. You, *Arithmetic version of Anderson localization via reducibility*, Preprint.

TALKS OF SESSION D33

MEAN LIKE MAPS GENERATED BY PATHOLOGICAL FUNCTIONS

Pál Burai
University of Debrecen, Hungary

The goal of this talk is the investigation of quasi-arithmetic expressions (close relatives of quasi-arithmetic means) generated by invertible (not necessary continuous) functions.

The resulted class can contain maps, which are not means, and which are not regular. However, it contains the whole class of quasi-arithmetic means.

References

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[5] Z. Daróczy and L. Losonczi, *Über die Erweiterung der auf einer Punktmenge additiven Funktionen*, Publ. Math. Debrecen **14** (1967), 239-245.

[6] W. Jabłoński, *Additive involutions and Hamel bases*, Aequationes Math. **89(3)** (2015), 575-582.

[7] M. Kuczma, *An introduction to the theory of functional equations and inequalities*, Birkhäuser Verlag, Basel, second edition, 2009. Cauchy's equation and Jensen's inequality, Edited and with a preface by Attila Gilányi.

[8] I. Makai, *Über invertierbare Lösungen der additiven Cauchy-Funktionalgleichung*, Publ. Math. Debrecen **16** (1969), 239-243.

APPLICATIONS OF FIXED POINT RESULTS TO THE GENERALIZED HYERS-ULAM STABILITY OF A FUNCTIONAL EQUATION

Liviu Cădariu-Brăiloiu
Politehnica University of Timisoara, Romania

In the last time there are emphasised several methods which allows to obtain Hyers-Ulam stability results for large classes of functional, differential and integral equations, in various spaces. For example, some fixed points theorems for operators (not necessarily linear) satisfying suitable very general properties have been proved recently. These results were used to obtain properties of generalized Hyers-Ulam stability, hyperstability, superstability, best constant, for different classes of functional equations.

The aim of this talk is to present an application of such a fixed point theorem for proving generalized Hyers-Ulam stability properties of a functional equation.

References

[1] J. Brzdęk, J. Chudziak, Z. Páles, *A fixed point approach to stability of functional equations*, Nonlinear Analysis - TMA **74** (2011), 6728–6732.

[2] J. Brzdęk, L. Cădariu, K. Ciepliński, *Fixed point theory and the Ulam stability*, J. Function Spaces **2014** (2014), Article ID 829419, 16 pp.

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[4] K. Ciepliński, *Applications of fixed point theorems to the Hyers-Ulam stability of functional equations – a survey*, Ann. Funct. Anal. **3(1)** (2012), 151–164.

ON SOME APPLICATIONS OF QUASIDEVIATION MEANS

Jacek Chudziak
University of Rzeszów, Poland

We show that willingness to accept (WTA) and willingness to pay (WTP) are particular cases of quasideviation means, introduced in [2]. Using this fact and applying some results in [3], we investigate the properties of WTA and WTP related to the experimentally observed disparity between them [1, 4].

References

[1] J.L. Knetsch, J.A. Sinden, *Willingness to pay and compensation demanded: experimental evidence of an unexpected disparity in measures of value*, The Quarterly Journal of Economics **99** (1984), 507-521.

[2] Zs. Páles, *Characterization of quasideviation means*, Acta. Math. Sci. Hungar. **40** (1982), 243-260.

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ON THE ASYMPTOTIC BEHAVIOUR OF THE ZEROS OF THE SOLUTIONS OF THE PANTOGRAPH EQUATION

Gregory Derfel
Ben-Gurion University of the Negev, Israel
Joint work with **Peter Grabner and Robert Tichy**.

We study asymptotic behaviour of the solutions of the pantograph equation. From this we derive asymptotic formula for the zeros of these solutions.

References

[1] C. Zhang, *An asymptotic formula for the zeros of the deformed exponential function*, J. Math. Anal. Appl. **441(2)** (2016), 565-573.

[2] L. Wang, C. Zhang, *Zeros of the deformed exponential function*, Advances in Mathematics **332** (2018), 311-348.

[3] G. Derfel, P. Grabner and R. Tichy, *On the asymptotic behaviour of the zeros of the solutions of a functional-differential equation with rescaling*, Operator Theory: Advances and Applications **263** (2018), 281-295.

NEW CHARACTERIZATIONS OF HYPERBOLICITY FOR LINEAR COCYCLES

Davor Dragičević
University of Rijeka, Croatia
Joint work with **Adina Luminita Sasu and Bogdan Sasu**.

We will describe some new characterizations of stability, expansivity and hyperbolicity of linear cocycles developed in [1] which are based on the ideas from subadditive ergodic theory.

References

[1] D. Dragičević, A.L. Sasu, B. Sasu, *On the asymptotic behavior of discrete dynamical systems - an ergodic theory approach*, submitted.

NEW INEQUALITIES FOR PROBABILITY FUNCTIONS IN THE TWO-PERSON RED-AND-BLACK GAME

Włodzimierz Fechner
Lodz University of Technology, Poland

We discuss a model of a two-person, non-cooperative stochastic game, inspired by the discrete version of the red-and-black gambling problem presented by Dubins and Savage [3]. Assume that two players hold certain amounts of money. At each stage of the game they simultaneously bid some part of their current fortune and the probability of winning or loosing depends on their bids. In many models of the red-and-black game it is assumed that the win probability is a function of the quotient of the bid of the first player and the sum of both bids. In the literature some additional properties, like concavity or super-multiplicativity, are assumed in order to ensure that bold and timid strategy is the Nash equilibrium (e.g. in works of Chen and Hsiau [1, 2]). In the talk we propose a generalization in which the probability of winning is a two-variable function which depends on both bids. We introduce two new functional inequalities whose solutions lead to win probability functions for which a Nash equilibrium is realized by the bold-timid strategy.

References

[1] M.R. Chen, S.R. Hsiau, *Two-person red-and-black games with bet-dependent win probability functions*, J. Appl. Probab. **43(4)** (2006), 905-915.

[2] M.R. Chen, S.R. Hsiau, *wo new models for the two-person red-and-black game*, J. Appl. Probab. **47(1)** (2010), 97-108.

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[4] W. Fechner, *New inequalities for probability functions in the two-person red-and-black game*, arXiv:1811.00359 [math.PR].

BASIC FUNCTIONS ON HYPERGROUP-TYPE STRUCTURES

Żywilla Fechner
Lodz University of Technology, Poland
Joint work with **László Székelyhidi**.

The aim of the talk is to present a characterization of functions like exponential monomials, polynomials and moment functions. We are interested in functions defined on some special type of hypergroups like affine groups, double coset hypergroups and hypergroup joins. We also discuss a connection of these functions with spectral synthesis problems.

References

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EMBEDDABILITY OF PAIRS OF WEIGHTED QUASI-ARITHMETIC MEANS INTO A SEMIFLOW

Dorota Głazowska
University of Zielona Góra, Poland
Joint work with **Justyna Jarczyk and Witold Jarczyk**.

Let $I \subset \mathbb{R}$ be an interval. Given any continuous strictly monotonic function $f : I \to \mathbb{R}$ and $p \in (0,1)$ the formula

$$A_p^f(x,y) = f^{-1}\left(pf(x) + (1-p)f(y)\right),$$

defines a mean on I called the *quasi-arithmetic mean generated by f and weighted by p.*

We determine the form of all semiflows of pairs of weighted quasi-arithmetic means, those over positive dyadic numbers as well as those continuous ones. Then the iterability of such pairs is characterized: necessary and sufficient conditions for a given pair of weighted quasi-arithmetic means to be embeddable into a continuous semiflow are given. In particular, it turns out that surprisingly the existence of a square iterative root in the class of such pairs implies the embeddability.

GAUSSIAN ALGORITHM FOR MAPPINGS BUILT OF PARAMETRIZED MEANS

Justyna Jarczyk
University of Zielona Góra, Poland

After recalling the notion of iterate of a function depending on a parameter, introduced by K. Baron and M. Kuczma in [1], we present a counterpart of Gaussian algorithm for mappings built of parametrized means. We consider also a special case of the so-called random means and describe some specific properties of the limit of their Gauss iterates.

References

[1] K. Baron, M. Kuczma, *Iteration of random-valued functions on the unit interval*, Colloq. Math. **37** (1977), 263-269.

GENERALIZED GAUSSIAN ALGORITHM

Witold Jarczyk
John Paul II Catholic University of Lublin, Poland

The classical Gaussian algorithm runs as follows: taking any $x, y \in (0, +\infty)$ put $x_1 := x$, $y_1 := y$ and

$$x_{n+1} := A(x_n, y_n), \quad y_{n+1} := G(x_n, y_n), \qquad n \in \mathbb{N},$$

where A and G state for the arithmetic and geometric mean, respectively. Gauss proved that both the sequences converge to a common limit, say $A \otimes G(x, y)$. The function $A \otimes G$ is a mean on $(0, +\infty)$, i.e. it satisfies

$$\min\{x, y\} \leq A \otimes G(x, y) \leq \max\{x, y\}, \qquad x, y \in (0, +\infty),$$

and has nice properties.

Iterating the map $(A, G) : (0, +\infty)^2 \to (0, +\infty)^2$ one can write down the convergence of Gaussian iterates to $A \otimes G$ as

$$(A, G)^i \to (A \otimes G, A \otimes G).$$

The Gauss procedure has been fairly extended to a pretty large class of pairs (M, N) of means on an arbitrary interval I. The talk is a survey of results concerning the convergence of iterates $(M, N)^i$ and properties of the mean

$$M \otimes N := \lim_{i \to \infty} (M, N)^i.$$

Starting with ideas and results aggregated by J.M. Borwein and P.B. Borwein from different papers more than 30 years ago we come to those proved by J. Matkowski.

FUNCTIONAL EQUATIONS, FIELD HOMOMORPHISMS AND DERIVATIONS IN THE LIGHT OF SPECTRAL THEORY

Gergely Kiss
Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Hungary
Joint work with **Eszter Gselmann and Csaba Vincze**.

First, in my talk I discuss the solutions of linear functional equations on fields of the form

$$\sum_{i=1}^{n} f_i(b_i x + c_i y) = 0 \qquad \forall x, y \in K,$$

where b_i, c_i are given constants, K is the field and f_i are unknown functions. I present that typically the set of solutions is a linear space containing field homomorphisms and higher order derivations. This result is based on spectral synthesis. Here I recall the theoretic background and discuss the main tools that we of use. In the second part of my presentation I study functional equations

$$\sum_{i=1}^{n} f_i^{p_i}(x^{q_i}) = 0 \qquad \forall x \in K,$$

and

$$\sum_{i=1}^{n} x^{p_i} f_i(x^{q_i}) = 0 \qquad \forall x \in K,$$

where p_i, q_i are positive integers and f_i are additive functions, that characterize field homomorphisms and higher order derivations, respectively. Among other techniques these results deliberately use spectral theory. Finally, I mention some further directions of research in this area. My talk is based on the [1, 2, 3].

References

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[3] E. Gselmann, G. Kiss, Cs. Vincze, *Characterization of field homomorphisms through Pexiderized functional equations*, Journal of difference equations and applications **25** (2019), 26 pp.

ON FRACTIONAL ITERATES OF A BROUWER HOMEOMORPHISM

Zbigniew Leśniak
Pedagogical University of Cracow, Poland

We present a method for finding continuous (and consequently homeomorphic) orientation preserving iterative roots of a Brouwer homeomorphism for which there exists a family of pairwise disjoint invariant lines covering the plane.

To obtain the roots we use properties of the equivalence classes of the codivergency relation. In particular, the key role plays the fact that each of the invariant lines of the considered family is contained either in the set of regular points or in the set of irregular points of the given Brouwer homeomorphism.

References

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[2] Z. Leśniak, *On properties of the set of invariant lines of a Brouwer homeomorphism*, J. Difference Equ. Appl. **24** (2018), 746-752.

ON BETWEENNESS-PRESERVING MAPPINGS

Janusz Morawiec
University of Silesia in Katowice, Poland
Joint work with **Wiesław Kubiś and Thomas Zürcher**.

We are interested in a Euclidean version of betweenness. We say that a point z is between two points x and y if and only if z is in the convex hull of x and y. In this setting, we call a betweenness-preserving map monotone. The aim of this talk is to present regularity results for monotone mappings in the plane.

SOME RESULTS FOR CONJUGATE EQUATIONS

Kazuki Okamura
Shinshu University, Japan

I will talk about conjugate maps between two iterated function systems driven by several weak contractions, depending on [3] and [4]. Specifically, a special case of our framework is as follows: Let X and Y be compact metric spaces. Let I be a finite set. Assume that for each $i \in I$, weak contractions $f_i : X \to X$ and $g_i : Y \to Y$ are given. Consider the solution $\varphi : X \to Y$ satisfying that

$$\varphi(f_i(x)) = g_i(\varphi(x)),\ i \in I, x \in X. \tag{1}$$

Conjugate equations of this kind are a certain generalization of de Rham's functional equations [5]. They are considered by Zdun [8], Girgensohn-Kairies-Zhang [2], Shi-Yilei [7], Serpa- Buescu [6] and Bárány-Kiss-Kolossváry [1].

I will mainly talk about regularity of a unique solution φ of (1). Then, I will give examples to which our results are applicable. If time is permitted, I will also discuss existence and uniqueness of a more general class of conjugate equations than above.

References

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COMMUTING RATIONAL FUNCTIONS REVISITED

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Let A and B be rational functions on the Riemann sphere. The classical Ritt theorem states that if A and B commute and do not have an iterate in common, then up to a conjugacy they are either powers, or Chebyshev polynomials, or Lattès maps. This result however provides no information about commuting rational functions which *do* have a common iterate. On the other hand, non-trivial examples of such functions exist and were constructed already by Ritt. In the talk we present new results concerning this class of commuting rational functions. In particular, we describe a method which permits to describe all rational functions commuting with a given rational function.

ON DERIVATIONS WITH ADDITIONAL PROPERTIES

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A function $d : \mathbb{R} \to \mathbb{R}$ is called a *derivation* if, for all $x, y \in \mathbb{R}$,

$$d(x+y) = d(x) + d(y) \qquad \text{and} \qquad d(xy) = yd(x) + xd(y).$$

It is a nontrivial fact that, for any non-algebraic number $t \in \mathbb{R}$, there exists a derivation which does not vanish at t. Nonzero derivations have many striking applications in the theory of functional equations and functional inequalities. Derivations derivate many of the elementary functions. For instance, if $f : I \to \mathbb{R}$ is the ratio of two polynomials with algebraic coefficients, then, for every $x \in I$,

$$d(f(x)) = f'(x)d(x).$$

It has been an old problem of the theory of functional equations whether there exists a nonzero derivation which derivates the exponential function or any of the trigonometric functions in the above sense. Our main result shows that the answer to this problem is affirmative.

QUASI-ARITHMETIC GAUSS-TYPE ITERATION

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For a sequence of continuous, monotone functions $f_1, \dots, f_n \colon I \to \mathbb{R}$ (I is an interval) we define the mapping $M \colon I^n \to I^n$ as a Cartesian product of quasi-arithmetic means generated by f_j-s, that is functions $A^{[f_j]}(v_1, \dots, v_n) := f_j^{-1}\big(\frac{1}{n}(f_j(v_1) + \dots + f_j(v_n))\big)$. It is known that, for every initial vector, the iteration sequence of this mapping tends to the diagonal of I^n.

We prove that whenever all f_j-s are $\mathcal{C}^2$ with nowhere vanishing first derivative, then this convergence is quadratic. We present both qualitative- and quantitative-type results concerning this iteration. In particular, we deliver an effective upper estimation of the value $\operatorname{Var} M^k(v)$ and calculate the limit $\frac{\operatorname{Var} M^{k+1}(v)}{(\operatorname{Var} M^k(v))^2}$ in a nondegenerated case.

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ON FUNCTIONAL EQUATIONS CHARACTERIZING GENERALIZED POLYNOMIALS

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We present some results on solving functional equations that characterize generalized polynomials. The results are essentially coming from our earlier works but we are going to investigate some new problems. We will give a description of solutions defined on Abelian groups and ask about the possible solutions in the case where semigroups (Abelian) are considered.

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FUNCTIONAL EQUATIONS VIA SPECTRAL SYNTHESIS

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Spectral synthesis studies the structure of translation invariant spaces of continuous functions over topological groups. The masterpiece is Laurent Schwartz's theorem stating that on the real line every translation invariant linear space of continuous complex valued functions which is closed under compact convergence is the closure of all exponential polynomials included in the space. As the solution space of a great variety of systems of convolution type functional equations satisfies these conditions spectral synthesis can be applied to describe the solutions. These ideas are worth for generalizations to obtain extensions of classical results of abstract harmonic analysis for functions without growth conditions (boundedness, integrability, etc.) Recently extensions of Schwartz's theorem have been proved over discrete groups using ring-theoretical methods, and spherical versions of the theorem have been obtained on spaces of functions invariant under various subgroups of the general linear group. Also extensions of the basic results to more general situations have been proved by relaxing the group-structure. In

this survey talk we present the fundamental methods, ideas and results together with relevant applications to functional equations.

A RING-THEORETIC APPROACH TO DISCRETE SPECTRAL SYNTHESIS

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Joint work with **László Székelyhidi**.

Let G be an Abelian group and let $\mathcal{C}(G)$ be the vector space of complex-valued functions on G. With the topology of pointwise convergence, $\mathcal{C}(G)$ is a locally convex space. The group G acts on $\mathcal{C}G$ by translations. Closed submodules of $\mathcal{C}G$ are called *varieties*. Consider the bilinear product $\mathcal{C}(G) \times \mathbb{C}G \to \mathbb{C}$ given by

$$\langle \sum_{x \in G} a_x x,\, f \rangle = \sum_{x \in G} a_x f(x).$$

Assigning the function $f \mapsto \langle a,\, f \rangle$ to a in $\mathbb{C}G$ yields an identification of $\mathbb{C}GG$ with the space of linear functionals on $\mathcal{C}G$ that are continuous with respect to the topology of pointwise convergence. Assigning the map $a \mapsto \langle a,\, f \rangle$ to f provides an identification of $\mathcal{C}(G)$ with the algebraic dual $\mathbb{C}G^*$. Defining orthogonal complements in the usual way, the Hahn-Banach theorem yields that the map $V \mapsto V^{\perp}$ is a one-to-one correspondence between varieties and ideals of $\mathbb{C}G$.

We exploit this to investigate to characterise the dual $\mathbb{C}G/V^{\perp}$ when V is a variety with *spectral analysis* - each subvariety of V contains a one-dimensional module - that is *syntheziable* - topologically generated by its finite-dimensional subvarieties- or possesses *spectral synthesis* - only has synthesizable subvarieties. We shall see that these properties correspond to well-known and well-researched properties of commutative rings. If V has spectral synthesis- the strongest of the listed properties, then $\mathbb{C}G/V^{\perp}$ emerges as "almost" Noetherian. Finally, we discuss how the ring theoretic results may be used to provide a description of the module structure of V.

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INVARIANT MANIFOLDS WITH/WITHOUT SPECTRAL GAP

Weinian Zhang
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In this talk we discuss invariant manifolds obtained with or without a spectral gap condition, showing approximation to weak hyperbolic manifolds (with gap condition) and giving the existence and smoothness for invariant submanifolds on a center manifold (without gap condition).

SMOOTH LINEARIZATION WITH A NONUNIFORM DICHOTOMY

Wenmeng Zhang
Chongqing Normal University, China
Joint work with **Davor Dragičević and Weinian Zhang**.

In this talk, we give two smooth linearization theorems for $C^{1,1}$ nonautonomous systems with a nonuniform strong exponential dichotomy. The first theorem concerns C^1 linearization with a

gap condition, while the second one concerns simultaneously differentiable and Hölder continuous linearization without any gap conditions. Restricted in the autonomous case, the second result gives the simultaneously differentiable and Hölder linearization of $C^{1,1}$ hyperbolic systems without any non-resonant conditions.

NONUNIFORM EXPONENTIAL DICHOTOMY AND ADMISSIBILITY

Linfeng Zhou
Sichuan University, China
Joint work with **Kening Lu and Weinian Zhang**.

Nonuniform exponential dichotomy describes nonuniform hyperbolicity for linear nonautonomous dynamical systems. In this talk, we present results on the relationships between nonuniform exponential dichotomies and admissible pairs for classes of weighted bounded functions, and the equivalent relationships between nonuniform exponential dichotomy and admissible pairs of classes of Lyapunov bounded functions.

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