DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D34

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CONTENTS

Plenary lectures **7**

Artur Avila, GENERIC CONSERVATIVE DYNAMICS 7

Alessio Figalli, ON THE REGULARITY OF STABLE SOLUTIONS TO SEMI-LINEAR ELLIPTIC PDES 7

Martin Hairer, RANDOM LOOPS 8

Stanislav Smirnov, 2D PERCOLATION REVISITED 8

Shing-Tung Yau, STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY 8

Maciej Zworski, FROM CLASSICAL TO QUANTUM AND BACK 9

Public lecture **11**

Alessio Figalli, FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY 11

Invited talks of part D3 **13**

Luigi Chierchia, KAM THEORY FOR SECONDARY TORI 13

Rafael de la Llave, SOME GEOMETRIC MECHANISMS FOR ARNOLD DIFFUSION 14

Desmond Higham, DIFFERENTIAL EQUATIONS FOR NETWORK CENTRALITY 14

Hinke Osinga, ROBUST CHAOS: A TALE OF BLENDERS, THEIR COMPUTATION, AND THEIR DESTRUCTION 15

Vladimir Protasov, THE JOINT SPECTRAL RADIUS AND FUNCTIONAL EQUATIONS: A RECENT PROGRESS 16

Emmanuel Trélat, OPTIMAL CONTROL AND APPLICATIONS TO AEROSPACE 17

Warwick Tucker, SMALL DIVISORS AND NORMAL FORMS 17

Walter Van Assche, ORTHOGONAL POLYNOMIALS AND PAINLEVÉ EQUATIONS 18

Sjoerd Verduyn Lunel, DELAY EQUATIONS AND TWIN SEMIGROUPS 18

Jiangong You, DYNAMICAL SYSTEM APPROACH TO SPECTRAL THEORY OF QUASI-PERIODIC SCHRÖDINGER OPERATORS 19

Talks of session D34 **21**

Martin Berz, RIGOROUS INTEGRATION OF FLOWS OF ODES USING TAYLOR MODELS 21

Renato Calleja, TORUS KNOT CHOREOGRAPHIES IN THE N-BODY PROBLEM 22

Maciej Capiński, ARNOLD DIFFUSION IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM 22

Hayato Chiba, A BIFURCATION OF THE KURAMOTO MODEL ON NETWORKS 23

Jacek Cyranka, CONTRACTIBILITY OF A PERSISTENCE MAP PREIMAGE . 24

Zbigniew Galias, ENCLOSURE OF THE DOUBLE SCROLL ATTRACTOR FOR THE CHUA'S CIRCUIT WITH A CUBIC NONLINEARITY 25

Marian Gidea, SYMBOLIC DYNAMICS AND STOCHASTIC BEHAVIOR IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM 26

Marcel Guardia, DIFFUSIVE BEHAVIOR ALONG MEAN MOTION RESONANCES IN THE RESTRICTED 3 BODY PROBLEM 27

Àlex Haro, SINGULARITY THEORY FOR KAM TORI: FROM SYMPLECTIC GEOMETRY TO APPLICATIONS THROUGH ANALYSIS 28

Bernd Krauskopf, A HETERODIMENSIONAL CYCLE IN A 4D FLOW 28

Hieronim Kubica, PERSISTENCE OF NORMALLY HYPERBOLIC INVARIANT MANIFOLDS IN THE ABSENCE OF RATE CONDITIONS 29

Ezequiel Maderna, HYPERBOLIC EXPANSIONS WITH ARBITRARY LIMIT SHAPE 30

Kyoko Makino, RIGOROUS GLOBAL SEARCH, DETERMINATION OF MANIFOLDS AND THEIR HOMOCLINIC POINTS, AND ENTROPY ESTIMATES 31

Jason D. Mireles James, VALIDATED NUMERICS FOR STABLE/UNSTABLE MANIFOLDS OF DELAY DIFFERENTIAL EQUATIONS 32

Markus Neher, INTERVAL AND TAYLOR MODEL METHODS FOR ODES . . . 32

Gabriella Pinzari, RENORMALIZABLE INTEGRABILITY OF THE PARTIALLY AVERAGED NEWTONIAN POTENTIAL 33

Pablo Roldán, CONTINUATION OF PERIODIC SOLUTIONS FROM THE CLASSICAL TO THE CURVED THREE-BODY PROBLEM 34

Marco Sansottera, ANALYTIC STUDY OF THE SECULAR DYNAMICS OF EXOPLANETARY SYSTEMS 35

Tere M. Seara, ON THE BREAKDOWN OF SMALL AMPLITUDE BREATHERS FOR THE REVERSIBLE KLEIN-GORDON EQUATION 36

Bruno Vergara, CONVEXITY OF WHITHAM'S HIGHEST WAVE 37

Adrian Weisskopf, NORMAL FORM METHODS AND RIGOROUS GLOBAL OPTIMIZATION FOR THE ASTRODYNAMICAL BOUNDED MOTION PROBLEM 37

Daniel Wilczak, CONTINUATION AND BIFURCATIONS OF HALO ORBITS IN THE CIRCULAR RESTRICTED THREE BODY PROBLEM 39

Piotr Zgliczyński, CENTRAL CONFIGURATIONS IN PLANAR n-BODY PROBLEM FOR $n = 5, 6, 7$ WITH EQUAL MASSES 40

Ke Zhang, DIFFUSION LIMIT FOR THE SLOW-FAST STANDARD MAP 41

PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D3

KAM THEORY FOR SECONDARY TORI

Luigi Chierchia
Roma Tre University, Italy
Joint work with **Luca Biasco**.

As well known, classical KAM (Kolmogorov, Arnold, Moser) theory deals with the persistence, under small perturbations, of real-analytic (or smooth) Lagrangian tori for nearly-integrable non-degenerate Hamiltonian systems. In this talk I will present a new *uniform* KAM theory apt to deal also with secondary tori, i.e., maximal invariant tori (with different homotopy) "generated" by the perturbation (and that do not exist in the integrable limit). The word "uniform" means that primary and secondary tori are constructed simultaneously; in particular, in the case of Newtonian mechanical systems on $\mathbf{T}^d$, it is proven that, for generic perturbations, the union of primary and secondary tori leave out a region of order $\varepsilon|\log\varepsilon|^a$, if ε is the norm of the perturbation, in agreement (up to the logarithmic correction) with a conjecture by Arnold, Kozlov and Neishtadt.

Some of these results have been announced in the note [1].

References

[1] L. Biasco, L. Chierchia, *On the measure of Lagrangian invariant tori in nearly-integrable mechanical systems*, Rend. Lincei Mat. Appl. **26** (2015), 423-432.

SOME GEOMETRIC MECHANISMS FOR ARNOLD DIFFUSION

Rafael de la Llave
Georgia Institute of Technology, USA

We consider the problem whether small perturbations of integrable mechanical systems can have very large effects. Since the work of Arnold in 1964, it is known that there are situations where the perturbations can accumulate. This can be understood by noting that the small perturbations generate some invariant structures that, with their stable and unstable manifolds can cover a large region in phase space. We will present recent developments in identifying these invariant objects, both in finite and in infinite dimensions.

DIFFERENTIAL EQUATIONS FOR NETWORK CENTRALITY

Desmond Higham
University of Edinburgh, UK

I will derive and discuss two circumstances where ODEs arise in the study of large, complex networks. In both cases, the overall aim is to identify the most important nodes in a network-this task is useful, for example, in digital marketing, security and epidemiology. In one case, we define our node centrality measure using the concept of nonbacktracking walks. This requires us to derive an expression for an exponential-type generating function associated with the walk counts of different length. Solving the ODE leads to a computationally useful characterisation of the centrality measure. In another case, we are presented with a time-ordered sequence of networks; for example, recording who emailed who over each one-minute time-window. Here, by considering the asymptotic limit as the window size tends to zero, we arrive at a limiting ODE that may be treated with a numerical method. Results for both algorithms will be illustrated on real network examples.

ROBUST CHAOS: A TALE OF BLENDERS, THEIR COMPUTATION, AND THEIR DESTRUCTION

Hinke Osinga
University of Auckland, New Zealand
Joint work with **Stephanie Hittmeyer, Bernd Krauskopf, and Katsutoshi Shinohara**.

A blender is an intricate geometric structure of a three- or higher-dimensional diffeomorphism [1]. Its characterising feature is that its invariant manifolds behave as geometric objects of a dimension that is larger than expected from the dimensions of the manifolds themselves. We introduce a family of three-dimensional Hénon-like maps and study how it gives rise to an explicit example of a blender [2, 3]. We employ our advanced numerical techniques to present images of blenders and their associated one-dimensional stable manifolds. Moreover, we develop an effective and accurate numerical test to verify what we call the *carpet property* of a blender. This approach provides strong numerical evidence for the existence of the blender over a large parameter range, as well as its disappearance and geometric properties beyond this range. We conclude with a discussion of the relevance of the carpet property for chaotic attractors.

References

[1] C. Bonatti, S. Crovisier, L.J. Díaz, A. Wilkinson, *What is... a blender?*, Not. Am. Math. Soc. **63** (2016), 1175-1178.

[2] L.J. Díaz, S. Kiriki, K. Shinohara, *Blenders in centre unstable Hénon-like families: with an application to heterodimensional bifurcations*, Nonlinearity **27** (2014), 353-378.

[3] S. Hittmeyer, B. Krauskopf, H.M. Osinga, K. Shinohara, *Existence of blenders in a Hénon-like family: geometric insights from invariant manifold computations*, Nonlinearity **31** (2018), R239-R267.

THE JOINT SPECTRAL RADIUS AND FUNCTIONAL EQUATIONS: A RECENT PROGRESS

Vladimir Protasov
University of L'Aquila, Italy & Lomonosov Moscow State University, Russia

Joint spectral radius of matrices have been used since late eighties as a measure of stability of linear switching dynamical systems. Nearly in the same time it has found important applications in the theory of refinement equations (linear difference equations with a contraction of the argument), which is a key tool in the construction of compactly supported wavelets and of subdivision schemes in approximation theory and design of curves and surfaces. However, the computation or even estimation of the joint spectral radius is a hard problem. It was shown by Blondel and Tsitsiklis that this problem is in general algorithmically undecidable. Nevertheless recent geometrical methods [1,2,3,4] make it possible to efficiently estimate this value or even find it precisely for the vast majority of matrices. We discuss this issue and formulate some open problems.

References

[1] N. Guglielmi, V.Yu. Protasov, *Exact computation of joint spectral characteristics of matrices*, Found. Comput. Math **13** (2013), 37-97.

[2] C. Möller, U. Reif, *A tree-based approach to joint spectral radius determination*, Linear Alg. Appl. **563** (2014), 154-170.

[3] N. Guglielmi, V.Yu. Protasov, *Invariant polytopes of linear operators with applications to regularity of wavelets and of subdivisions*, SIAM J. Matrix Anal. **37** (2016), 18-52.

[4] T. Mejstrik, *Improved invariant polytope algorithm and applications*, arXiv:1812.03080.

OPTIMAL CONTROL AND APPLICATIONS TO AEROSPACE

Emmanuel Trélat
Sorbonne Université, France

I will report on nonlinear optimal control theory and show how it can be used to address problems in aerospace, such as orbit transfer. The knowledge resulting from the Pontryagin maximum principle is in general insufficient for solving adequately the problem, in particular due to the difficulty of initializing the shooting method. I will show how the shooting method can be successfully combined with numerical homotopies, which consist of deforming continuously a problem towards a simpler one. In view of designing low-cost interplanetary space missions, optimal control can also be combined with dynamical system theory, using the nice dynamical properties around Lagrange points that are of great interest for mission design.

SMALL DIVISORS AND NORMAL FORMS

Warwick Tucker
Uppsala University, Sweden
Joint work with **Zbigniew Galias**.

In this talk, we will discuss the computational challenges of computing trajectories of a non-linear ODE in a region close to a saddle-type fixed-point. By introducing a carefully selected close to identity change of variables, we can bring the non-linear ODE into an "almost" linear system. This normal form system has an explicit transfer-map that transports trajectories away from the fixed point in a controlled manner. Determining the domain of existence for such a change of variables poses some interesting computational challenges. The proposed method is quite general, and can be extended to the complex setting with spiral saddles. It is also completely constructive which makes it suitable for practical use. We illustrate the use of the method by a few examples.

ORTHOGONAL POLYNOMIALS AND PAINLEVÉ EQUATIONS

Walter Van Assche
KU Leuven, Belgium

Painlevé equations are nonlinear differential equations for which the branch points do not depend on the initialcondition (no movable branch points). There are also discrete Painlevé equations which are non-linearrecurrence relations with enough structure (symmetry and geometry) that make them integrable.Both the discrete and continuous Painlevé equations appear in a natural way in the theory of orthogonalpolynomials. The recurrence coefficients of certain families of orthogonal polynomials often satisfya discrete Painlevé equation. The Toda equations describing the movement of particles with an exponentialinteraction with their neighbors, is equivalent to an exponential modification $e^{xt}\,d\mu(x)$ of the orthogonality measure $d\mu$ for a family of orthogonal polynomials, and the corresponding recurrencecoefficients satisfy the Toda equations, which is a system of differential-difference equations. Combining this with the discrete Painlevé equations then gives a Painlevé differential equation.We will illustrate this by a number of examples. The relevant solutions of these Painlevé equationsare usually in terms of known special functions, such as the Bessel functions, the Airy function, parabolic cylinder functions, or (confluent) hypergeometric functions.

References

[1] W. Van Assche, *Orthogonal Polynomials and Painlevé Equations*, Australian Mathematical Society Lecture Notes **27**, Cambridge University Press, (2018).

DELAY EQUATIONS AND TWIN SEMIGROUPS

Sjoerd Verduyn Lunel
Utrecht University, Netherlands
Joint work with **Odo Diekmann**.

A delay equation is a rule for extending a function of time towards the future on the basis of the (assumed to be) known past. By translation along the extended function (i.e., by updating the history), one defines a dynamical system. If one chooses as state-space the continuous initial functions, the translation semigroup is continuous, but the initial data corresponding to the fundamental solution is not contained in the state space.

In ongoing joint work with Odo Diekmann, we choose as state space the space of bounded Borel functions and thus sacrifice strong continuity in order to gain a simple description of the variation-of-constants formula.

The aim of the lecture is to introduce the perturbation theory framework of twin semigroups on a norming dual pair of spaces, to show how renewal equations fit in this framework and to sketch how neutral equations can be covered. The growth of an age-structured population serves as a pedagogical example.

DYNAMICAL SYSTEM APPROACH TO SPECTRAL THEORY OF QUASI-PERIODIC SCHRÖDINGER OPERATORS

Jiangong You
Nankai University, China

The spectral theory of quasiperiodic operators is a fascinating field which continuously attracts a lot of attentions for its rich background in quantum physics as well as its rich connections with many mathematical theories and methods. In this talk, I will briefly introduce the problems in this field and their connections with dynamical system. I will also talk about some recent results joint with Avila, Ge, Leguil, Zhao and Zhou on both spectrum and spectral measure by reducibility theory in dynamical systems.

References

[1] A. Avila, J. You and Q. Zhou, *Sharp phase transitions for the almost Mathieu operator*, Duke Math. J. **166** (2017), 2697-2718.

[2] A. Avila, J. You and Q. Zhou, *The dry Ten Martini problem in the non-critical case*, Preprint.

[3] M. Leguil, J. You, Z. Zhao, Q. Zhou, *Asymptotics of spectral gaps of quasi-periodic Schrödinger operators*, arXiv:1712.04700.

[4] L. Ge, J. You and Q. Zhou, *Exponential dynamical localization: Criterion and applications*, arXiv:1901.04258.

[5] L. Ge, J. You, *Arithmetic version of Anderson localization via reducibility*, Preprint.

TALKS OF SESSION D34

RIGOROUS INTEGRATION OF FLOWS OF ODES USING TAYLOR MODELS

Martin Berz
Michigan State University, USA
Joint work with **Kyoko Makino**.

Taylor models combine the advantages of numerical methods and their efficiency under tightly controlled computational resources even for complex problems, with the advantages of symbolic approaches and their ability to be rigorous and to allow the treatment of functional dependencies instead of merely points. The result is a local representation of a function by its Taylor expansion and a mathematically rigorous bound for the approximation error.

The resulting differential algebraic calculus comprising an algebra with differentiation and integration is particularly amenable to the study of ODEs and PDEs based on fixed point problems from functional analysis. We describe the development of rigorous tools to determine enclosures of flows of general nonlinear differential equations based on Picard iterations.

The methods can be used for the computation of enclosures of flows over large domains to prescribed accuracy via domain decomposition methods. We study the behavior of the methods for several dynamical systems, and in particular analyze suitable parameter settings to efficiently balance local domain size versus local order. Comparisons to other recently proposed computational approaches are given, showing the advantages of the Taylor model methods for large domains.

TORUS KNOT CHOREOGRAPHIES IN THE N-BODY PROBLEM

Renato Calleja
National Autonomous University of Mexico, Mexico
Joint work with **Eusebius Doedel, Carlos García Azpeitia, Jason Mireles-James, and Jean-Philippe Lessard**.

N-body choreographies are periodic solutions to the N-body equations in which N equal masses chase each other around a fixed closed curve. In this talk I will present a systematic approach for proving the existence of spatial choreographies in the gravitational N body problem with the help of the digital computer. These arise from the polygonal system of N bodies in a rotating frame of reference. In rotating coordinates, after exploiting the symmetries, the equation of a choreographic configuration is reduced to a delay differential equation (DDE) describing the position and velocity of a single body. We prove that a dense set of Lyapunov orbits, with frequencies satisfying a Diophantine equation, correspond to choreographies.

References

[1] R. Calleja, E. Doedel, and C. García-Azpeitia, *Symmetries and choreographies in families that bifurcate from the polygonal relative equilibrium of the n-body problem*, Celestial Mech. Dynam. Astronom. **130** (2018), 130:48.

[2] R. Calleja, C. García-Azpeitia, J.P. Lessard, and J.D. Mireles-James, *Torus knot choreographies in the n-body problem*,Preprint, available at http://cosweb1.fau.edu/ jmireles-james/torusKnotChoreographies.html

ARNOLD DIFFUSION IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM

Maciej Capiński
AGH University of Science and Technology, Poland
Joint work with **Marian Gidea**.

We present a topological mechanism for diffusion in Hamiltonian systems and apply it to the Planar Elliptic Restricted Three-Body Problem. We treat the elliptic problem as a perturbation of the circular problem, where the perturbation parameter ε is the eccentricity of the orbits of the primaries. We measure the energy as the Hamiltonian H of the circular problem. Our objective is to prove that for any $\varepsilon > 0$ there exist orbits which start with some value $H = h$ and finish with $H = h + c$, where $c > 0$ is independent from ε. Our method is based on topological shadowing of trajectories along homoclinic intersections of invariant manifolds. We perform a geometric construction, which allows us to obtain orbits for which we can control the increase in energy. The method is suitable for computer assisted proofs and can be used to obtain explicit bounds on the energy changes, for explicit ranges of the perturbation parameter. We apply it to the setting of the Neptune-Tryton system.

The construction also leads to symbolic dynamics in energy and to stochastic properties of the diffusing orbits. This will be the subject of the talk by Marian Gidea in the same session.

A BIFURCATION OF THE KURAMOTO MODEL ON NETWORKS

Hayato Chiba
Tohoku University, Japan
Joint work with **Georgi Medvedev**.

For the mean-field limit of a system of globally coupled phase oscillators defined on networks, a bifurcation from the incoherent state to the partially locked state at the critical coupling strength is investigated based on the generalized spectral theory. This reveals that a network topology affects the dynamics through the eigenvalue problem of a certain Fredholm integral operator which defines the structure of a network.

References

[1] H. Chiba, G.S. Medvedev, *The mean field analysis for the Kuramoto model on graphs I. The mean field equation and transition point formulas*, Discret. Contin. Dyn. S.-A **39** (2019), 131-155.

[2] H. Chiba, G.S. Medvedev, *The mean field analysis ofthe Kuramoto model on graphs II. Asymptotic stability of the incoherentstate, center manifold reduction, and bifurcations*, Series, Discret. Contin. Dyn. S.-A, (2019).

[3] H. Chiba, G.S. Medvedev, M.S. Muzuhara, *Bifurcations in the Kuramoto model on graphs*, Chaos **28** (2019), 073109.

CONTRACTIBILITY OF A PERSISTENCE MAP PREIMAGE

Jacek Cyranka
University of California, San Diego, USA & University of Warsaw, Poland
Joint work with **Konstantin Mischaikow**.

This work is motivated by the following question in data-driven study of dynamical systems: given a dynamical system that is observed via time series of persistence diagrams that encode topological features of solutions snapshots, what conclusions can be drawn about solutions of the original dynamical system? In this paper we provide a definition of a persistence diagram for a point in $\mathbb{R}^N$ modeled on piecewise monotone functions. We then provide conditions under which time series of persistence diagrams can be used to guarantee the existence of a fixed point of the flow on $\mathbb{R}^N$ that generates the time series. To obtain this result requires an understanding of the preimage of the persistence map. The main theorem of this paper gives conditions under which these preimages are contractible simplicial complexes.

References

[1] J. Cyranka and K. Mischaikow, *Contractibility of a persistence map preimage*, arXiv e-prints 2018, arXiv:1810.12447.

ENCLOSURE OF THE DOUBLE SCROLL ATTRACTOR FOR THE CHUA'S CIRCUIT WITH A CUBIC NONLINEARITY

Zbigniew Galias
AGH University of Science and Technology, Poland
Joint work with **Warwick Tucker**.

We consider the Chua's circuit with a cubic nonlinearity. The dynamics of the circuit is defined by

$$C_1\dot{x}_1 = (x_2 - x_1)/R - g(x_1), \quad C_2\dot{x}_2 = (x_1 - x_2)/R + x_3, \quad L\dot{x}_3 = -x_2 - R_0x_3,$$

where $g(x_1) = g_1x_1 + g_2x_1^3$. The system is considered with the following parameter values $C_1 = 0.7$, $C_2 = 7.8$, $L = 1.891$, $R_0 = 0.01499$, $g_1 = -0.59$, $g_2 = 0.02$, and $R = 2$, for which in simulations one observes the double scroll attractor.

Let us define $\Sigma = \Sigma_1 \cup \Sigma_2$, where $\Sigma_1 = \{x\colon x_1 = 2.1647\}$ and $\Sigma_2 = \{x\colon x_1 = -2.1647\}$. The *return map* $P : \Sigma \mapsto \Sigma$ is defined as $P(x) = \varphi(\tau(x), x)$, where $\varphi(t, x)$ is the trajectory based at x, and $\tau(x)$ is the *return time* after which the trajectory $\varphi(t, x)$ returns to Σ.

A candidate $T \subset \Sigma$ for a trapping region enclosing the numerically observed attractor of the return map P is constructed. The return map P is not defined on the whole set T. This is a consequence of the fact that the double scroll attractor contains the origin-an unstable equilibrium. For some initial points in $x \in T$ the corresponding trajectories converge to the origin, i.e. $\varphi(t, x) \to (0, 0, 0)$ for $t \to \infty$. It follows that standard rigorous integration procedures cannot be used to study the dynamics of the system over the whole set T. A method to handle trajectories passing arbitrarily close to an equilibrium is needed. Such trajectories may have arbitrarily large return times. The Jacobian matrix J at the origin has one real positive eigenvalue $\lambda \approx 0.2066$ and a pair of complex eigenvalues with negative real parts $\alpha \pm \beta i \approx -0.075 \pm 0.1966$. Normal form theory is used to develop a method to find enclosures of trajectories in a neighborhood of an unstable fixed point of a spiral type.

We prove the existence of a trapping region enclosing the double scroll attractor for the Chua's circuit with a cubic nonlinearity. More precisely, we prove the following theorem: for each $x \in T$ either $P(x) \in T$ or the trajectory $\varphi(t, x)$ converges to the origin without intersecting Σ, i.e., $\varphi(t, x) \to (0, 0, 0)$ for $t \to \infty$ and $\{\varphi(t, x)\colon t > 0\} \cap \Sigma = \emptyset$.

In the computer assisted proof, to handle trajectories passing close to the origin, we define the cylinder C centered at the origin. We also define the entry set, which is a part of the cylinder side and the exit set consisting of two parts each enclosed in one of the cylinder bases.

The proof of the main results is composed of three parts. In the first part the set T is covered by boxes. For each box we prove that either the image of this box under P is enclosed in T or all trajectories based in this box enter the cylinder C through the entry set. In the second part, we show that all trajectories based at the exit set reach T. In the third part of the proof, we show that trajectories based at the entry set either converge to the origin or exit the cylinder through the exit set.

The first two parts of the proof are carried out using the CAPD library for the computation of trajectories and the evaluation of the return map P. The third part of the proof is carried out using the normal form theory.

SYMBOLIC DYNAMICS AND STOCHASTIC BEHAVIOR IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM

Marian Gidea
Yeshiva University, USA
Joint work with **Maciej Capiński**.

We study Hamiltonian instability in the elliptic restricted three-body problem, in the context of a concrete model, on the motion of a small body (e.g., asteroid or spaceship) relative the Neptune-Triton system.

The elliptic restricted three-body problem can be regarded as a perturbation of the circular restricted three-body problem, with the perturbation parameter ε being the eccentricity of the orbits of the primaries. When the perturbation parameter is set to zero, the total energy H_ε of the system is preserved. When the perturbation parameter is non-zero, the total energy may vary. We provide two global instability results concerning the variation of energy along trajectories.

First, we show that for every suitably small, non-zero perturbation parameter, there exist trajectories along which the energy makes chaotic jumps. That is, given a sequence of energy level sets $(h^\sigma)_{\sigma\geq 0}$, with $\|h^{\sigma+1} - h^\sigma\| > 2\eta$, for some suitable $\eta > 0$, there exists a trajectory with $\|H_\varepsilon(t^\sigma) - h^\sigma\| < \eta$, for some times $t^\sigma > 0$ and all $\sigma \geq 0$.

Second, we show that the distributions of energies along orbits starting from some sets of initial conditions converge to a Brownian motion with drift as the perturbation parameter tends to zero. Moreover, we can obtain any desired values of the drift and of the variance for the limiting Brownian motion, for appropriate sets of initial conditions. That is, if we consider the stochastic process $X_t^\varepsilon(z)$ representing the evolution of the energy along a trajectory starting from some point z, with appropriately rescaled time t, then, for every $\mu, \sigma \in \mathbb{R}$ there exists a set Ω_ε of initial points z for which $X_t^\varepsilon - X_0^\varepsilon$ converges in distribution to $\mu t + \sigma W_t$ as $\varepsilon \to 0$, where W_t is the standard Wiener process.

In both cases we obtain an explicit range of the perturbation parameter ε for which the above phenomena occur. The proof of the results is based on topological methods and validated numerics.

Our results address conjectures made by Arnold [1] and Chirikov [2].

References

[1] V.I. Arnold, *Instability of dynamical systems with several degrees of freedom*, Sov. Math. Doklady **5** (1964), 581-565.

[2] B. Chirikov, *A universal instability of many-dimensional oscillator systems*, Phys. Rep. **52** (1979), 264-379.

DIFFUSIVE BEHAVIOR ALONG MEAN MOTION RESONANCES IN THE RESTRICTED 3 BODY PROBLEM

Marcel Guardia
Polytechnic University of Catalonia, Spain
Joint work with **Vadim Kaloshin, Pau Martín, and Pau Roldán**.

Consider the Restricted Planar Elliptic Three Body Problem. This problem models the Sun-Jupiter-Asteroid dynamics. For eccentricity of Jupiter e_0 small enough we show that there exists a family of probability measures ν_{e_0} supported at the $3:1$ mean motion resonance such that the pushforward under the associated Hamiltonian flow has the following property. At the time scale te_0^{-2}, the distribution of the eccentricity of the Asteroid weakly converges to an (Ito stochastic) diffusion process on the line as $e_0 \to 0$. This resonance corresponds to the biggest of the Kirkwood gap on the Asteroid belt in the Solar System.

SINGULARITY THEORY FOR KAM TORI: FROM SYMPLECTIC GEOMETRY TO APPLICATIONS THROUGH ANALYSIS

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Joint work with **Rafael de la Llave and Alejandra González**.

We present a method to find nontwist KAM tori. These are tori for which the twist condition fails. Our method also leads to a natural classification of KAM tori which is based on Singularity Theory. This talk aims to illustrate the main ideas of our approach, going from rigorous results to numerical computations up to the verge of breakdown.

A HETERODIMENSIONAL CYCLE IN A 4D FLOW

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Joint work with **Andy Hammerlindl, Gemma Mason and Hinke M. Osinga**.

A heterodimensional cycle consists of a pair of heteroclinic connections between two saddle periodic orbits with unstable manifolds of different dimensions. Recent theoretical work for diffeomorphisms of dimension at least three has shown that the existence of heterodimensional cycles may be a C^1-robust property. We study a concrete example of a heterodimensional cycle in a flow, specifically in a four-dimensional Atri model of intracellular calcium dynamics. For suitable parameter values, this model has two saddle periodic orbits of different index. We employ a boundary-value problem setup to compute their global invariant manifolds to show that and how they intersect in a connecting orbit of codimension one and an entire cylinder of connecting orbits. We present the different invariant objects in different projections of the four-dimensional phase space, as well as in intersection with a three-dimensional Poincaré section. In this way, we examine how this heterodimensional cycle arises and organises the nearby dynamics.

PERSISTENCE OF NORMALLY HYPERBOLIC INVARIANT MANIFOLDS IN THE ABSENCE OF RATE CONDITIONS

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Joint work with **Maciej Capiński**.

Normally hyperbolic invariant manifolds (NHIMs) [8] are persistent [2, 3, 4, 5]. The converse is also true - persistent manifolds are precisely the normally hyperbolic ones [9]. These results take advantage of analytical assumptions. Features like rate conditions associated with the map driving the dynamics, smooth structure on the invariant set, or C^1 topology on the space of admissible perturbations play a key role. In a recent result, we prove that a weaker form of persistence is viable also in a setting which relies only on topological, qualitative assumptions about the dynamics [1].

We consider perturbations of normally hyperbolic invariant manifolds, under which they can loose their hyperbolic properties. We show that if the perturbed map which drives the dynamical system preserves the properties of topological expansion and contraction, described in terms of covering relations [6, 7], then the manifold is perturbed to an invariant set. The main feature is that our results do not require the rate conditions to hold after the perturbation. In this case the manifold can be perturbed to an invariant set, which is not a topological manifold. The method used to show this is not itself perturbative. It can be applied to establish the existence of invariant sets within a prescribed neighborhood also in the absence of a normally hyperbolic invariant manifold prior to perturbation. The dynamics is assumed to be given by a continuous map, without the assumption of invertibility.

References

[1] M. Capiński, H. Kubica, *Persistence of normally hyperbolic invariant manifolds in the absence of rate conditions*, presented for publication, 2018

[2] N. Fenichel, *Persistence and smoothness of invariant manifolds for flows*, Indiana Univ. Math. J. **21** (1971/1972), 193–226.

[3] N. Fenichel, *Asymptotic stability with rate conditions*, Indiana Univ. Math. J. **23** (1973/74), 1109–1137.

[4] N. Fenichel, *Asymptotic stability with rate conditions for dynamical systems*, Bull. Amer. Math. Soc. **80** (1974), 346–349.

[5] N. Fenichel, *Asymptotic stability with rate conditions II*, Indiana Univ. Math. J. **26** (1977), 81–93.

[6] M. Gidea, P. Zgliczyński, *Covering relations for multidimensional dynamical systems*, J. Differential Equations **202** (2004), 32–58.

[7] M. Gidea, P. Zgliczyński, *Covering relations for multidimensional dynamical systems II*, J. Differential Equations **202** (2004), 59–80.

[8] M. Hirsch, C. Pugh, M. Shub *Invariant Manifolds*, Lecture Notes in Mathematics, Springer, Berlin Heidelberg, 2006.

[9] R. Mañé, *Persistent manifolds are normally hyperbolic*, Trans. Amer. Math. Soc. **246** (1978), 261–283.

HYPERBOLIC EXPANSIONS WITH ARBITRARY LIMIT SHAPE

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Joint work with **Andrea Venturelli**.

A well-known fact in the classical N-body problem is that if we normalize by the size of the configuration a completely parabolic motion, then the normalized configuration converge to the set of central configurations.

We will show that there is no such restriction for motions with positive energy. Moreover, we will show the existence of hyperbolic motions with arbitrarily chosen limit shape, and this for any given initial configuration of the bodies. The energy level $h > 0$ of the motion can also be chosen arbitrarily. The proof uses variational methods and represents a new application of Marchal's theorem, whose main use in recent literature has been to prove the existence of periodic orbits.

RIGOROUS GLOBAL SEARCH, DETERMINATION OF MANIFOLDS AND THEIR HOMOCLINIC POINTS, AND ENTROPY ESTIMATES

Kyoko Makino
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Joint work with **Martin Berz**.

Taylor models provide enclosures of functions over a domain within a relaxation band of their Taylor expansion around a point inside the domain. They are obtained automatically by evaluating the code list of the underlying function in Taylor model arithmetic, and under minimal requirements on the underlying floating point arithmetic, the enclosures are mathematically rigorous. The widths of the resulting band scales with a high order of the width of the domain, and so in practice enclosures are obtained that are usually much sharper than those from conventional rigorous methods like intervals, centered forms, and related linearizations for all but the simplest cases. In general, the complexity and nonlinearity of the underlying function dictates optimum order and domain widths to achieve a desired accuracy. The resulting rigorous relaxations can be used for the local description of objective functions and constraints in rigorous global search. Furthermore, the resulting representations can be used efficiently for higher order domain reduction based on conditions on the objective function and the constraints.

The methods can be applied to various problems in dynamical systems. First, Taylor models allow for the computation of tight enclosures of manifolds of dynamical systems. Once these enclosures are given, they can be used with the Taylor model-based rigorous global optimizer to find and isolate all homoclinic points. From these it is possible to determine so-called homoclinic tangles, which contain information on lower bounds of topological entropy of the underlying systems. Various examples of the practical use of the methods are given.

VALIDATED NUMERICS FOR STABLE/UNSTABLE MANIFOLDS OF DELAY DIFFERENTIAL EQUATIONS

Jason D. Mireles James
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Joint work with **Jean-Philippe Lessard and Olivier Hénot**.

The parameterization method is a general functional analytic framework for studying invariant manifolds. The idea is to formulate a chart or covering map for the manifold as the solution of an appropriate invariance equation. Studying the invariance equation leads to both numerical schemes for approximating the invariant manifold and to a posteriori methods for quantifying discretization and truncation errors. This talk considers the parameterization method for unstable manifolds of delay differential equations (DDEs), focusing on the numerical implementation as well as the derivation of mathematically rigorous computer assisted error bounds. One challenge is the fact that the solution of a DDE depends on both present and past states, so that a DDE generates an infinite dimensional dynamical system. The invariant manifolds studied here play an important role in describing the global dynamics of this system.

INTERVAL AND TAYLOR MODEL METHODS FOR ODES

Markus Neher
KIT, Germany

Verified integration methods for ODEs are methods that compute rigorous bounds for some specific solution or for the flow of some initial set of a given ODE. Interval arithmetic has been used for calculating such bounds for solutions of initial value problems. The origin of these methods dates back to Ramon Moore [2].

Unfortunately, interval methods sometimes suffer from overestimation. This can be caused by the *dependency problem*, that is the lack of interval arithmetic to identify different occurrences of the same variable, and by the *wrapping effect*, which occurs when intermediate results of a calculation are enclosed into intervals. In verified integration this happens when enclosures of the flow at intermediate time steps of the interval of integration are computed. Overestimation may then degrade the computed enclosure of the flow, enforce miniscule step sizes, or provoke premature abortion of the integration.

Taylor models, developed by Martin Berz in the 1990s, combine interval arithmetic with symbolic computations [1]. A Taylor model consists of a multivariate polynomial and a remainder interval. In all computations, the polynomial part is handled by symbolic calculations, which are essentially unaffected by the dependency problem or the wrapping effect. Only the interval remainder term and polynomial terms of high order, which are usually small, are bounded using interval arithmetic. Taylor models also benefit from their capability of representing non-convex sets. For nonlinear ODEs, this increased flexibility in admissible boundary curves for the flow is an intrinsic advantage over traditional interval methods.

In our talk, we analyze Taylor model mehods for the verified integration of ODEs and compare these methods with interval methods.

References

[1] M. Berz, *From Taylor series to Taylor models*, AIP Conference Proceedings **405** (1997), 1-23.

[2] R.E. Moore, *Interval Analysis*, Prentice Hall, Englewood Cliffs, N.J., 1966.

RENORMALIZABLE INTEGRABILITY OF THE PARTIALLY AVERAGED NEWTONIAN POTENTIAL

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Definition Let h, g be two (commuting) functions of the form

$$h(p,q,y,x) = \widehat{h}(\mathrm{I}(p,q),y,x)\ , \qquad g(p,q,y,x) = \widehat{g}(\mathrm{I}(p,q),y,x)$$

where

$$(p, q, y, x) \in \mathcal{D} := \mathcal{B} \times U$$

with $U \subset \mathbb{R}^2$, $\mathcal{B} \subset \mathbb{R}^{2n}$ open and connected, $(p, q) = (p_1, \cdots, p_n, q_1, \cdots, q_n)$ conjugate coordinates with respect to the two-form $= dy \wedge dx + \sum_{i=1}^n dp_i \wedge dq_i$ and $\mathrm{I}(p, q) = (\mathrm{I}_1(p, q), \cdots, \mathrm{I}_n(p, q))$, with

$$\mathrm{I}_i : \ \mathcal{B} \to \mathbb{R} \ , \qquad i = 1, \cdots, n$$

pairwise Poisson commuting:

$$\{\mathrm{I}_i, \mathrm{I}_j\} = 0 \qquad \forall 1 \le i < j \le n \qquad i + 1, \cdots, n.$$

We say that h is *renormalizably integrable via* g if there exists a function

$$\widetilde{h} : \qquad \mathrm{I}(\mathcal{B}) \times g(U) \to \mathbb{R} \ ,$$

such that

$$h(p, q, y, x) = \widetilde{h}(\mathrm{I}(p, q), \widehat{g}(\mathrm{I}(p, q), y, x))$$

for all $(p, q, y, x) \in \mathcal{D}$.

It is proved that the partial average i.e., the Lagrange average with respect to *just one* of the two mean anomalies, of the Newtonian part of the perturbing function in the three-body problem Hamiltonian is renormalizably integrable. Consequences on the dynamics of the three-body problem are briefly discussed. The talk is based on [1], ArXiv: arXiv:1808.07633 and work in progress.

References

[1] G. Pinzari, *A first integral to the partially averaged Newtonian potential of the three-body problem*, Celest Mech. Dyn. Astr. **131(22)** (2019).

CONTINUATION OF PERIODIC SOLUTIONS FROM THE CLASSICAL TO THE CURVED THREE-BODY PROBLEM

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Joint work with **Abimael Bengochea and Ernesto Perez-Chavela**.

In the classical 3-body problem it is known that any three masses lying on the vertices of an equilateral triangle generate a relative equilibria, which is a periodic solution. We will discuss the possible continuation of this periodic solution to the curved 3-body problem. (The curved problem is the extension of the classical one to a manifold of constant curvature.)

ANALYTIC STUDY OF THE SECULAR DYNAMICS OF EXOPLANETARY SYSTEMS

Marco Sansottera
University of Milan, Italy
Joint work with **Anne-Sophie Libert, Ugo Locatelli, and Antonio Giorgilli**.

The search for exoplanets around nearby stars has produced a massive amount of observational data, pointing out the peculiar character of the Solar system. To date, more than 600 multiple planet systems have been found and the number of discovered exoplanets with unexpected orbital properties (such as highly eccentric orbits, mutually inclined planetary orbits, hot Jupiters, compact multiple systems) constantly increases.

The Laplace-Lagrange secular theory uses the circular approximation as a reference, thus its applicability to extrasolar systems can be doubtful. In this talk we aim to show that perturbation theory reveals very efficient for describing the long-term evolution of extrasolar systems.

First we study the long-term evolution of coplanar extrasolar systems with two planets by extending the Laplace-Lagrange theory (see [1, 2]). We identify three categories of systems: (i) *secular systems*, whose long-term evolution is accurately described using high order expansions in the eccentricities; (ii) *near a mean-motion resonance systems*, for which an approximation at order two in the masses is required; (iii) *really close to or in a mean-motion resonance systems*, for which a resonant model has to be used.

Then, being the inclinations of exoplanets detected via radial velocity method essentially unknown, we show how perturbation theory can be used in order to provide estimations of the ranges of mutual inclinations that are compatible with the long-term stability of the system. We propose a novel procedure (see [3]): a *reverse KAM approach* by using normal forms depending on a free parameter related to the unknown mutual inclinations of the exoplanets. Our approach can interestingly complement the concept of *AMD-stability* (see [4, 5]) to analyze the dynamics of the multiple-planet extrasolar systems.

References

[1] A.-S. Libert, M. Sansottera, *On the extension of the Laplace-Lagrange secular theory to order two in the masses for extrasolar systems*, Celest. Mech. Dyn. Astr. **117** (2013), 149–168.

[2] M. Sansottera, A.-S. Libert, *Resonant Laplace-Lagrange theory for extrasolar systems in mean-motion resonance*, Celest. Mech. Dyn. Astr., to appear (2019).

[3] M. Volpi, U. Locatelli, M. Sansottera, *A reverse KAM method to estimate unknown mutual inclinations in exoplanetary systems*, Celest. Mech. Dyn. Astr. **130(36)** (2018).

[4] J. Laskar, A.C. Petit, *AMD-stability and the classification of planetary systems*, Astron. & Astroph. **605** (2017), A72.

[5] A.C. Petit, J. Laskar, G. Boué, *AMD-stability in the presence of first-order mean motion resonances*, Astron. & Astroph., **607** (2017), A35.

ON THE BREAKDOWN OF SMALL AMPLITUDE BREATHERS FOR THE REVERSIBLE KLEIN-GORDON EQUATION

Tere M. Seara
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Joint work with **Marcel Guardia and Otavio Gomide**.

Breathers are periodic in time spatially localized solutions of evolutionary PDEs. They are known to exist for the sine-Gordon equation but are believed to be rare in other Klein-Gordon equations. Exchanging the roles of time and position, breathers can be interpreted as homoclinic solutions to a steady solution. In this talk, I will explain how to obtain an asymptotic formula for the distance between the stable and unstable manifold of the steady solution when the steady solution has weakly hyperbolic one dimensional stable and unstable manifolds. Their distance is exponentially small with respect to the amplitude of the breather and therefore classical perturbative techniques cannot be applied.

CONVEXITY OF WHITHAM'S HIGHEST WAVE

Bruno Vergara
ICMAT, Spain
Joint work with **Alberto Enciso and Javier Gómez-Serrano**.

In this talk I will discuss a conjecture of Ehrnström and Wahlén [1] on the profile of solutions of extreme form to Whitham's model of shallow water waves. This is a non-local dispersive equation featuring travelling waves and singularities. Analogously to Stokes waves for Euler, we will see that there exists a highest, cusped and periodic solution to this model which is convex between consecutive crests [2].

References

[1] M. Ehrnström, E. Wahlén, *On Whitham's conjecture of a highest cusped wave for a nonlocal shallow water wave equation*, Ann. Inst. H. Poincaré Anal. Non. Linéaire, in press (2019), arXiv:1602.05384.

[2] A. Enciso, J. Gómez-Serrano, B. Vergara, *Convexity of Whitham's highest cusped wave*, (2018), arXiv:1810.10935.

NORMAL FORM METHODS AND RIGOROUS GLOBAL OPTIMIZATION FOR THE ASTRODYNAMICAL BOUNDED MOTION PROBLEM

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Joint work with **Martin Berz and Roberto Armellin**.

Due to their common origin and mathematical underpinnings, it is sometimes possible to transfer specific advanced methods to analyze dynamical systems from one field of applications to another. In this work, we illustrate the transfer of differential algebra (DA) based normal form methods and rigorous global optimization for Nekhoroshev-type stability estimates, which were first developed in the field of particle beam physics and accelerator physics, to the field of astrodynamics to design bounded motion in the Earth's zonal problem.

The DA framework [1] and in particular the DA normal form algorithm [4], and their associated techniques are hybrid methods of numerical and analytic calculations and have been established by Berz *et al.* The methods can be turned mathematically rigorous by fully accounting for expansion errors and floating point inaccuracies, as is done in the Taylor model methods discussed in this session.

Many of the DA tools have been applied in the field of accelerator physics, where they reveal details of those dynamical systems that are otherwise very difficult to obtain by conventional methods. More recently, researchers have begun on the fruitful transfer of those DA methods to the astrodynamics community [3, 6, 5, 2]. An advancement of this transfer to normal form methods provides new possibilities, as it will be demonstrated in this work, including the capability of determining entire sets of bounded motion orbits in the full zonal problem.

Given an origin preserving Poincaré return map of a repetitive Hamiltonian system expanded in its phase space coordinates and system parameters, the DA normal form algorithm provides a nonlinear change of variables by an order-by-order transformation to normal form coordinates in which the motion is rotationally invariant. This circular phase space behavior allows for a straightforward extraction of the phase space rotation frequency and an action-angle parameterization of the normal form motion.

The normal form parameterization is used to find orbit sets satisfying the bounded motion condition, i.e. same average nodal period and drift in the ascending node of the bounded orbits. For the right Poincaré surface of the Poincaré return map, the inverse normal form transformation is used to parameterize the map by the action-angle parameterization of the corresponding normal form motion. Averaging the map over a full phase space revolution by a path integral along the angle-parameterization yields the averaged nodal period and drift in the ascending node for which the bounded motion conditions are straightforwardly imposed. Sets of highly accurate bounded orbits are obtained, extending over several thousand kilometers and valid for more than ten years.

References

[1] M. Berz, *Differential algebraic description of beam dynamics to very high orders*, Part. Accel. **24** (1988), 109-124.

[2] A. Wittig, R. Armellin, *High order transfer maps for perturbed Keplerian motion*, Celestial Mechanics and Dynamical Astronomy **122** (2015), 333-358.

[3] P. Di Lizia, R. Armellin, M. Lavagna, *Application of high order expansions of two-point boundary value problems to astrodynamics*, Celestial Mechanics and Dynamical Astronomy **102** (2008), 355-375.

[4] M. Berz, *Modern Map Methods in Particle Beam Physics*, Academic Press, 1999.

[5] R. Armellin, P. Di Lizia, F. Topputo, M. Lavagna, F. Bernelli-Zazzera, M. Berz, *Gravity assist space pruning based on differential algebra*, Celestial Mechanics and Dynamical Astronomy **106** (2010), 1-24.

[6] R. Armellin, P. Di Lizia, M. Berz, K. Makino, *Computing the critical points of the distance function between two Keplerian orbits via rigorous global optimization*, Celestial Mechanics and Dynamical Astronomy **107** (2010), 377-395.

CONTINUATION AND BIFURCATIONS OF HALO ORBITS IN THE CIRCULAR RESTRICTED THREE BODY PROBLEM

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Joint work with **Irmina Walawska**.

We propose a general framework for computer-assisted verifcation of isochoronus, period- tupling or touch-and-go bifurcations of symmetric periodic orbits for reversible maps. The framework is then adopted to Poincaré maps in reversible autonomous Hamiltonian systems.

In order to justify the applicability of the method, we study bifurcations of halo orbits in the Circular Restricted Three Body Problem. We give a computer-assisted proof [1] of the existence of wide branches of halo orbits bifurcating from $L_{1,2,3}$-Lyapunov families and for wide range of mass parameter. For two physically relevant mass parameters (Sun-Jupiter and Earth-Moon systems) we prove, that $L_{1,2}$ branches of halo orbits undergo multiple period doubling, quadrupling and third-order touch-and-go bifurcations.

The computer-assisted proof uses rigorous ODE solvers and algorithms for computation of Poincare maps and their derivatives from the CAPD library [2].

References

[1] I. Walawska, D. Wilczak, *Validated numerics for period-tupling and touch-and-go bifurcations of symmetric periodic orbits in reversible systems*, Commun. Non. Sci. Num. Simul. **74C** (2019), 0-54.

[2] CAPD library: C++ package for validated numerics for discrete and continuous dynamical systems, http://capd.ii.uj.edu.pl.

CENTRAL CONFIGURATIONS IN PLANAR n-BODY PROBLEM FOR $n = 5, 6, 7$ WITH EQUAL MASSES

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Joint work with **Małgorzata Moczurad**.

We give a computer assisted proof of the full listing of central configuration for n-body problem for Newtonian potential on the plane for $n = 5, 6, 7$ with equal masses. We show all these central configurations have a reflective symmetry with respect to some line. For $n = 8, 9, 10$ we establish the existence of central configurations without any reflectional symmetry.

References

[1] M. Moczurad, P. Zgliczyński, *Central configurations in planar n-body problem for $n = 5, 6, 7$ with equal masses*, arXiv:1812.07279.

DIFFUSION LIMIT FOR THE SLOW-FAST STANDARD MAP

Ke Zhang
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Joint work with **Alex Blumenthal and Jacopo De Simoi**.

We discuss a simple two-dimensional slow-fast system, which is conjugate to the Chirikov standard map with a large parameter. Consider a random initial condition and view the nth iterate of the slow variable as a sequence of random variables, we prove a central limit theorem for this sequence, under suitable parameter values and time horizon. Our main motivation for studying this model is a phenomenon called "scattering by resonance" in physical systems.