DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D41

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D4

MULTISCALE METHODS AND ANALYSIS FOR THE DIRAC EQUATION IN THE NONRELATIVISTIC LIMIT REGIME

Weizhu Bao
National University of Singapore, Singapore

In this talk, I will review our recent works on numerical methods and analysis for solving the Dirac equation in the nonrelativistic limit regime, involving a small dimensionless parameter which is inversely proportional to the speed of light. In this regime, the solution is highly oscillating in time and the energy becomes unbounded and indefinite, which bring significant difficulty in analysis and heavy burden in numerical computation [4]. We begin with four frequently used finite difference time domain (FDTD) methods and the time splitting Fourier pseudospectral (TSFP) method and obtain their rigorous error estimates in the nonrelativistic limit regime by paying particularly attention to how error bounds depend explicitly on mesh size and time step as well as the small parameter [3]. Then we consider a numerical method by using spectral method for spatial derivatives combined with an exponential wave integrator (EWI) in the Gautschi-type for temporal derivatives to discretize the Dirac equation [3]. Rigorous error estimates show that the EWI spectral method has much better temporal resolution than the FDTD methods for the Dirac equation in the nonrelativistic limit regime [3]. We find that the time-splitting spectral method performs super-resolution in temporal discretization when the Dirac equation has no magnetic potential [5]. Based on a multiscale expansion of the solution, we present a multiscale time integrator Fourier pseudospectral (MTI-FP) method for the Dirac equation and establish its error bound which uniformly accurate in term of the small dimensionless parameter [1]. Numerical results demonstrate that our error estimates are sharp and optimal. Finally, these methods and results are then extended to the nonlinear Dirac

equation in the nonrelativistic limit regime [2]. This is a joint work with Yongyong Cai, Xiaowei Jia, Qinglin Tang and Jia Yin.

References

[1] W. Bao, Y. Cai, X. Jia and Q. Tang, *A uniformly accurate multiscale time integrator pseudospectral method for the Dirac equation in the nonrelativistic limit regime*, SIAM J. Numer. Anal. **54** (2016), 1785-1812.

[2] W. Bao, Y. Cai, X. Jia and J. Yin, *Error estimates of numerical methods for the nonlinear Dirac equation in the nonrelativistic limit regime*, Sci. China Math. **59** (2016), 1461-1494.

[3] W. Bao, Y. Cai, X. Jia and Q. Tang, *Numerical methods and comparison for the Dirac equation in the nonrelativistic limit regime*, J. Sci. Comput. **71** (2017), 1094-1134.

[4] W. Bao and J. Yin, *A fourth-order compact time-splitting Fourier pseudospectral method for the Dirac equation*, Res. Math. Sci. **6** (2019), article 11.

[5] W. Bao, Y. Cai and J. Yin, *Improved stability of optimal traffic paths*, Super-resolution of time-splitting methods for the Dirac equation in the nonrelativistic limit regime, arXiv: 1811.02174.

ACCELERATED SIMULATION FOR PLASMA KINETICS

Russel Caflisch
New York University, USA
Joint work with **Denis Silantyev and Bokai Yann**.

This presentation will discuss the kinetics of Coulomb collisions in plasmas, as described by the Landau-Fokker-Planck equation, and its numerical solution using a Direct Simulation Monte Carlo (DSMC) method. Acceleration of this method is achieved by coupling the particle method to a continuum fluid description. Efficiency of the resulting hybrid method is greatly increased by inclusion of particles with negative weights. This complicates the simulation, and introduces difficulties have plagued earlier efforts to use negatively weighted particles. This talk will describe significant progress that has been made in overcoming those difficulties.

OPTIMAL SAMPLING AND RECONSTRUCTION IN HIGH DIMENSION

Albert Cohen
Sorbonne Université, France

Motivated by non-intrusive approaches for high-dimensional parametric PDEs, we consider the general problem of approximating an unknown arbitrary function in any dimension from the data of point samples. The approximants are picked from given or adaptively chosen finite dimensional spaces. One principal objective is to obtain an approximation which performs as good as the best possible using a sampling budget that is linear in the dimension of the approximating space. We will show that this object if can is met by taking a random sample distributed according to a well chosen probability measure, and reconstructing by appropriate least-squares or pseudo-spectral methods.

MARGINAL TRIVIALITY OF THE SCALING LIMITS OF CRITICAL ISING AND φ^4 MODELS IN 4D

Hugo Duminil-Copin
IHÉS, France & University of Geneva, Switzerland
Joint work with **Michael Aizenman**.

The question of constructing a non-Gaussian field theory, i.e. a field with non-zero Ursell functions, is at the heart of Euclidean (quantum) field theory. While non-triviality results in $d < 4$ and triviality results in $d > 4$ were obtained in famous papers by Glimm, Jaffe, Aizenman, Frohlich and others, the crucial case of dimension 4 remained open. In this talk, we show that any continuum φ^4 theory constructed from Reflection Positive lattice φ^4 or Ising models is inevitably free in dimension 4. The proof is based on a delicate study of intersection properties of a non-Markovian random walk appearing in the random current representation of the model.

FROM WIGNER-DYSON TO PEARCEY: UNIVERSAL EIGENVALUE STATISTICS OF RANDOM MATRICES

László Erdős
Institute of Science and Technology, Austria

E. Wigner's revolutionary vision postulated that the local eigenvalue statistics of large random matrices are independent of the details of the matrix ensemble apart from its basic symmetry class. There have recently been a substantial development to prove Wigner's conjecture for larger and larger classes of matrix ensembles motivated by applications. They include matrices with entries with a general correlation structure and addition of deterministic matrices in a random relative basis. We also report on three types of universality, commonly known as the bulk, edge and cusp universality, referring to the behaviour of the density of states in the corresponding energy regime. While bulk and edge universalities have been subject to intensive research, the cusp universality has been studied only in very special cases before. Our recent work settles the question of this third and last type of universality in full generality.

DISSIPATIVE SOLUTIONS TO THE COMPRESSIBLE EULER SYSTEM

Eduard Feireisl
Czech Academy of Sciences, Czech Republic
Joint work with **Dominic Breit and Martina Hofmanová**.

We introduce the concept of (generalized) dissipative solutions to the compressible Euler system and review their basic properties:

• **Existence.** Dissipative solutions exist globally in time for any finite energy initial data.

• **Maximal dissipation, semigroup selection.** One can select a solution semigroup among dissipative solutions. The selected solution maximizes the energy dissipation (entropy production), see [1].

• **Weak-strong uniqueness.** A dissipative and a weak solution emanating from the same initial data coincide as soon as the weak solution belongs to certain Besov class and its velocity gradient satisfies a one sided Lipschitz condition, see [2].

• **Convergence of numerical schemes.** Cesaro avarages produced by suitable numerical schemes converge strongly to a dissipative solution, see [3].

References

[1] D. Breit, E. Feireisl, M. Hofmanová, *Solution semiflow to the isentropic Euler system*, Arxive Preprint Series, arXiv 1901.04798, 2019.

[2] E. Feireisl, S.S. Ghoshal, A. Jana, *On uniqueness of dissipative solutions to the isentropic Euler system*, Arxive Preprint Series, arXiv 1903.11687, 2019.

[3] E. Feireisl, M. Lukáčová–Medviďová, H. Mizerová, *$\mathcal{K}$-convergence as a new tool in numerical analysis*, Arxive Preprint Series, arXiv 1904.00297, 2019.

FINITE DIMENSIONAL STATE REPRESENTATION OF STRUCTURED POPULATION MODELS

Mats Gyllenberg
University of Helsinki, Finland

Structured population models can be formulated as delay systems. We consider the question of when delay systems, which are intrinsically infinite dimensional, can be represented by finite dimensional systems. Specifically, we give conditions for when all the information about the solutions of the delay system can be obtained from the solutions of a finite system of ordinary differential equations. For linear autonomous systems and linear systems with time-dependent input we give necessary and sufficient conditions and in the nonlinear case we give sufficient conditions. The ideas and results are illustrated by models for infectious diseases and physiologically structured populations.

QUANTITATIVE LINEAR STABILITY (HYPOCOERCIVITY) FOR CHARGED PARTICLES IN A CONFINING FIELD

Clément Mouhot
University of Cambridge, UK
Joint work with **K. Carrapatoso, J. Dolbeault, F. Hérau, S. Mischler, and C. Schmeiser**.

We report on recent joint results in which we develop quantitative methods for proving the existence of a spectral gap and estimating the gap, for hypocoercive kinetic equations that combine the local conservation laws of fluid mechanics and a confining potential force. The proofs involve a cascade of correctors and global commutator estimates, as well as new quantitative inequalities of Korn type. The latter extend to the case of the whole space with a potential force the classical Korn inequality in a bounded domain of elasticity theory. These results are a step towards constructing global solutions near equilibrium to the full nonlinear Boltzmann equation for charged particles subject to a confining potential.

STOCHASTIC MODELING AND OPTIMIZATION IN HUMAN-MACHINE INTERACTION SYSTEMS

Thaleia Zariphopoulou
University of Texas at Austin, USA
Joint work with **Agostino Capponi and Svein Olefsson**.

I will introduce a family of human-machine interaction (HMI) models in optimal asset allocation, risk management and portfolio choice (robo-advising). Modeling difficulties stem from the limited ability to quantify the human's risk preferences and describe their evolution, but also from the fact that the stochastic environment, in which the machine optimizes, itself adapts to real-time incoming information that is exogenous to the human. Furthermore, the human's risk preferences and the machine's states may evolve at different scales. This interaction creates an

adaptive cooperative game with asymmetric and incomplete information exchange between the two parties.

As a result, challenging questions arise on, among others, how frequently the two parties should communicate, what information can the machine accurately detect, infer and predict, how the human reacts to exogenous events and what are the effects on the machine's actions, how to improve the inter-linked reliability between the human and the machine, and others.

Such HMI models give rise to new, non-standard optimization problems that include well-posed and ill-posed sub-problems, and combine adaptive stochastic control, stochastic differential games, optimal stopping, multi-scales and learning.

References

[1] A. Capponi, S. Olefsson and T. Zariphopoulou, *Personalized robo-advising*, Preprint, 2019.

TALKS OF SESSION D41

CORRELATION ENERGY OF THE MEAN-FIELD FERMI GAS BY THE METHOD OF COLLECTIVE BOSONIZATION

Niels Benedikter
Institute of Science and Technology, Austria
Joint work with **Phan Thành Nam, Marcello Porta, Benjamin Schlein, and Robert Seiringer**.

Quantum correlations play an important role in interacting systems; however, their mathematical description is a highly non-trivial task. I explain how correlations in fermionic systems can be described by bosonizing collective pair excitations. This leads us to an effective quadratic bosonic Hamiltonian. We establish a theory of approximate bosonized Bogoliubov transformations by which we derive a Gell-Mann-Brueckner-type formula for the fermionic ground state energy.

References

[1] N. Benedikter, P.T. Nam, M. Porta, B. Schlein, R. Seiringer, *Optimal Upper Bound for the Correlation Energy of a Fermi Gas in the Mean-Field Regime*, to appear in Communications in Mathematical Physics, 2019.

ON THE TYPE I BLOW-UP FOR THE INCOMPRESSIBLE EULER EQUATIONS

Dongho Chae
Chung-Ang University, South Korea
Joint work with **Jörg Wolf**.

In this talk we discuss the Type I blow up and the related problems in the 3D Euler equations. We say a solution v to the Euler equations satisfies Type I condition at possible blow up time T_* if $\limsup_{t\nearrow T_*}(T_*-t)\|\nabla v(t)\|_{L^\infty}<+\infty$. The scenario of Type I blow up is a natural generalization of the self-similar(or discretely self-similar) blow up. We present some recent progresses of our study regarding this. We first localize previous result that "small Type I blow up" is absent. After that we show that the atomic concentration of energy is excluded under the Type I condition. This result, in particular, solves the problem of removing discretely self-similar blow up in the energy conserving scale, since one point energy concentration is necessarily accompanied with such blow up. We also localize the Beale-Kato-Majda type blow up criterion. Using similar local blow up criterion for the 2D Boussinesq equations, we can show that Type I and some of Type II blow up in a region off the axis can be excluded in the axisymmetric Euler equations.

SUFFICIENT CONDITIONS FOR TURBULENCE SCALING LAWS IN 2D AND 3D

Michele Coti Zelati
Imperial College London, UK
Joint work with **Jacob Bedrossian, Sam Punshon-Smith, and Franziska Weber**.

We provide sufficient conditions for mathematically rigorous proofs of the third order universal laws for both 2d [1] and 3d [2] stochastically forced Navier-Stokes equations. These conditions, which we name weak anomalous dissipation, replace the classical anomalous dissipation condition. For statistically stationary solutions, weak anomalous dissipation appear to be very effective and not too far from being necessary as well.

References

[1] J. Bedrossian, M. Coti Zelati, S. Punshon-Smith, F. Weber, *Sufficient conditions for dual cascade flux laws in the stochastic 2d Navier-Stokes equations*, arXiv 1905.03299 (2019).

[2] J. Bedrossian, M. Coti Zelati, S. Punshon-Smith, F. Weber, *A Sufficient Condition for the Kolmogorov 4/5 Law for Stationary Martingale Solutions to the 3D Navier-Stokes Equations*, Comm. Math. Phys. **367** (2019), 1045-1075.

PROPAGATORS ON CURVED SPACETIMES

Jan Dereziński

University of Warsaw, Poland

Joint work with **Daniel Siemssen and Adam Latosiński**.

Quantum Field Theory on curved spacetimes has many interesting links to various branches of mathematics, such as differential geometry, symplectic dynamics, partial differential equations, pseudodifferential calculus, symmetric spaces and operator theory. I will discuss some of these links.

LOOP EQUATIONS AND A PROOF OF ZVONKINE'S qr-ELSV FORMULA

Petr Dunin-Barkowski

National Research University Higher School of Economics, Russia

Joint work with **Reinier Kramer, Alexandr Popolitov, and Sergey Shadrin**.

The talk is devoted to the outlining of the proof of the 2006 Zvonkine's conjecture that expresses Hurwitz numbers with completed cycles in terms of intersection numbers with the

Chiodo classes via the so-called r-ELSV formula. In fact, this proof works in even a bit more general setting, namely it works for the qr-ELSV formula (which is the orbifold generalization of the r-ELSV formula), conjectured recently in [1]. The proof relies on expressing both the aforementioned Hurwitz and intersection numbers in terms of expansions of multi-point functions resulting from the application of the spectral curve topological recursion procedure on a particular spectral curve.

The talk is based on [2].

References

[1] R. Kramer, D. Lewanski, A. Popolitov, S. Shadrin, *Towards an orbifold generalization of Zvonkine's r-ELSV formula*, arXiv:1703.06725, 1–20.

[2] P. Dunin-Barkowski, R. Kramer, A. Popolitov, S. Shadrin, *Loop equations and a proof of Zvonkine's qr-ELSV formula*, arXiv:1905.04524, 1–17.

CONTROL OF EIGENFUNCTIONS ON NEGATIVELY CURVED SURFACES

Semyon Dyatlov
University of California, Berkeley & Massachusetts Institute of Technology, USA
Joint work with **Jean Bourgain, Long Jin, and Stéphane Nonnenmacher**.

Given an L^2-normalized eigenfunction with eigenvalue λ^2 on a compact Riemannian manifold (M, g) and a nonempty open set $\Omega \subset M$, what lower bound can we prove on the L^2-mass of the eigenfunction on Ω? The unique continuation principle gives a bound for any Ω which is exponentially small as $\lambda \to \infty$. On the other hand, microlocal analysis gives a λ-independent lower bound if Ω is large enough, i.e. it satisfies the geometric control condition.

This talk presents a λ-independent lower bound for any set Ω in the case when M is a negatively curved surface, or more generally a surface with Anosov geodesic flow. The proof uses microlocal analysis, the chaotic behavior of the geodesic flow, and a new ingredient from harmonic analysis called the Fractal Uncertainty Principle. Applications include control for Schrödinger equation and exponential decay of damped waves.

ON THE DERIVATION OF KINETIC WAVE EQUATIONS

Pierre Germain
New York University, USA

Consider a nonlinear dispersive equation with random initial data. In the appropriate regime, it is conjectured that its dynamics are described, after averaging over the random data, by a kinetic wave equation. I will present recent progress towards the proof of this conjecture.

FROM CFT TO QUANTUM CURVES AND SUPER-AIRY STRUCTURES

Leszek Hadasz
Jagiellonian University in Kraków, Poland
Joint work with **Vincent Bouchard, Paweł Ciosmak, Zbigniew Jaskólski, Masahide Manabe, Kento Osuga, Błażej Ruba, and Piotr Sułkowski**.

During the talk I will discuss how the methods and notions developed in the area of two-dimensional, quantum conformal field theory allow to solve efficiently some problems related to matrix models: finding families of quantum curves related to a given classical algebraic curve and constructing interesting examples of super-Airy structures (algebras of differential operators engineered to solve topological recursion relations).

References

[1] P. Ciosmak, L. Hadasz, Z. Jakólski, M. Manabe and P. Sułkowski, *From CFT to Ramond super-quantum curves*, JHEP **1805** 133 (2018).

[2] V. Bouchard, P. Ciosmak, L. Hadasz, K. Osuga, B. Ruba and P. Sułkowski, *Super Airy Structures*, work in progress.

LOWER BOUND ON THE HARTREE-FOCK ENERGY OF THE ELECTRON GAS

Christian Hainzl
Universität Tübingen, Germany
Joint work with **David Gontier and Mathieu Lewin**.

The Hartree-Fock ground state of the Homogeneous Electron Gas is never translation invariant, even at high densities. As proved by Overhauser, the free Fermi Gas is always unstable under the formation of spin or charge density waves. I present the first explicit bound on the energy gain due to the breaking of translational symmetry. Our bound is exponentially small at high density, which justifies posteriori the use of the non-interacting Fermi Gas as a reference state in the large-density expansion of the correlation energy of the Homogeneous Electron Gas. Our work sheds a new light on the Hartree-Fock phase diagram of the Homogeneous Electron Gas.

NON-LINEAR STABILITY OF THE SCHWARZSCHILD FAMILY OF BLACK HOLES

Gustav Holzegel
Imperial College London, UK
Joint work with **Mihalis Dafermos, Igor Rodnianski, and Martin Taylor**.

I will discuss the statement and the proof of the finite-codimension non-linear stability of the Schwarzschild family as solutions to the vacuum Einstein equations. The proof relies crucially on our previous work [1] on the linear stability of the Schwarzschild family and makes use of many analytical techniques developed over the years in the analysis of hyperbolic equations on black hole spacetimes, including control of the non-linearities of the Einstein equations in the radiation zone.

References

[1] M. Dafermos, G. Holzegel, I. Rodnianski, *The linear stability of the Schwarzschild solution to gravitational perturbations*, Acta Mathematica **222** (2019), 1-214.

ON THE MATHEMATICAL ANALYSIS OF SYNCHRONIZATION FOR THE TIME-DELAYED KURAMOTO OSCILLATORS

Chun-Hsiung Hsia
National Taiwan University, Taiwan
Joint work with **Chang-Yeol Jung, Bongsuk Kwon, and Yoshihiro Ueda**.

We investigate the synchronized collective behavior of the Kuramoto oscillators with time-delayed interactions and phase lag effect. Both the phase and frequency synchronization are in view. We first prove the frequency synchronization for both semi-delay and full-delay models with heterogeneous time-delays and phase lags. We also prove the complete and partial phase synchronization for both models with the uniform time-delay and phase lag. Our results show that the Kuramoto models incorporated with small variation of time-delays and/or phase lag effect still exhibit the synchronization. These support that the original Kuramoto model (i.e., no time-delays/phase lags) is qualitatively robust in the perturbation of time-delay and phase lag effects. We also present several numerical experiments supporting our main results.

References

[1] C.-H. Hsia, C.-Y. Jung, B. Kwon, *On the synchronization theory of Kuramoto oscillators under the effect of inertia*, Journal of Differential Equations **267** (2019), 742-775.

[2] C.-H. Hsia, C.-Y. Jung, B. Kwon, Y. Ueda, *Synchronization of Kuramoto oscillators with time-delayed interactions and phase lag effect*, preprint.

MULTI-BUBBLES FOR A CRITICAL WAVE EQUATION

Jacek Jendrej
CNRS & Université Paris 13, France
Joint work with **Yvan Martel**.

If a solution of an evolution partial differential equation resembles, near some point in space, a rescaled copy of a fixed profile, with the scale tending to zero in finite or infinite time, we say that a *bubble* is created at this point. Such behavior can be possible only if the rescaling preserves the energy of the profile, which is called the *energy-critical* setting.

We consider the focusing nonlinear wave equation in the energy-critical case, in space dimension 5. Given any finite set of K points in space, we construct a solution for which a bubble is created at each of these points in infinite time. The energy of the solution is equal to the energy of the profile multiplied by K, which means that no energy is radiated in the process. To our knowledge, we provide the first construction of bubbling at multiple points for a wave equation.

ON UNIQUENESS OF GENERALIZED NEWTONIAN FLOWS

Petr Kaplický
Charles University, Czech Republic
Joint work with **Miroslav Bulíček, Frank Ettwein, and Dalibor Pražák**.

In [3] a new model for fluid dynamics was suggested by O.A. Ladyzhenskaya. The inner properties of the fluid were described by the constitutive relation

$$\mathbb{S} = (1 + |\mathbb{D}|^{p-2})\mathbb{D}$$

where $p > 1$ was a given parameter - power-law index, $\mathbb{S}$ was the stress tensor and $\mathbb{D}$ the symmetric part of the velocity gradient. If $p = 2$ the model reduces to the Navier-Stokes model. If $p > 2$ one expects that the model exhibits better properties. It is indeed so. In particular, if $p \geq 11/5$, one is allowed to test weak formulation of the equations with the weak solution itself. Consequently, any weak solution satisfies energy equality. A further motivation for this model was uniqueness of weak solutions. Already in [3], it is established provided $p \geq 5/2$ or in case of smooth initial condition for $p \geq 12/5$. The range $p \in [11/5, 12/5)$ however remained untouched except the case of spatial periodic condition, for which one can improve spatial regularity.

I will present results on uniqueness of the weak solutions to a class of systems, including the one mentioned above, in three-dimensional setting subjected to the homogeneous Dirichlet boundary condition. Under the natural monotonicity, coercivity and growth condition on the Cauchy stress tensor expressed by a power index $p \geq 11/5$, some regularity of a solution with respect to time variable was established. Consequently, this information can be used for showing the uniqueness of the solution provided that the initial data are good enough for all power–law indices $p \geq 11/5$, see [1, 2]. Such a result was available for $p \geq 12/5$ and therefore the result extends the uniqueness to the whole range of p's for which the energy equality holds.

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NORM APPROXIMATION FOR MANY-BOSON QUANTUM DYNAMICS

Marcin Napiórkowski

University of Warsaw, Poland

Joint work with **Christian Brennecke, Phan Thành Nam, and Benjamin Schlein**.

Because of the complexity of the many-body Schrödinger equation, to gain insight into the properties of many-body quantum systems it is necessary to use effective theories.

In my talk, I will review recent advances [1, 2] in the derivation of effective equations that govern the dynamics of Bose-Einstein condensates.

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ON THE CORRELATION ENERGY OF INTERACTING FERMI GASES IN THE MEAN-FIELD REGIME

Marcello Porta
Universität Tübingen, Germany
Joint work with **Christian Hainzl, Felix Rexze, Niels Benedikter, Phan Thành Nam, Benjamin Schlein, and Robert Seiringer**.

In this talk I will discuss the ground state properties of a homogeneous, interacting Fermi gas, in the mean-field regime. I will focus on the correlation energy, defined as the difference between many-body and Hartree-Fock ground state energies. It is a long-standing open problem in mathematical physics to rigorously compute this quantity, for large quantum systems. I will present upper and lower bounds for the correlation energy, that are optimal in their dependence on the number of particles, and that agree for small interactions. The lower bound captures the corrections to the energy predicted by second-order perturbation theory; it is based on the combination of Bogoliubov theory and on correlation inequalities for the many-body interaction. The upper bound establishes the validity of the random-phase approximation as a rigorous upper bound to the ground state energy; it is based on a suitable choice of the trial state, and on a rigorous bosonization scheme.

COMPACTNESS OF SIGN-CHANGING SOLUTIONS TO SCALAR CURVATURE-TYPE EQUATIONS WITH BOUNDED NEGATIVE PART

Bruno Premoselli
Université libre de Bruxelles, Belgium
Joint work with **Jérôme Vétois**.

We consider the equation $\Delta_g u + hu = |u|^{2^*-2}u$ in a closed Riemannian manifold (M, g), where $h \in C^{0,\theta}(M)$, $\theta \in (0,1)$ and $2^* = \frac{2n}{n-2}$, $n := \dim(M) \geq 3$. We obtain a sharp compactness result on the sets of sign-changing solutions whose negative part is *a priori* bounded. We obtain this result under the conditions that $n \geq 7$ and $h < \frac{n-2}{4(n-1)}\mathrm{Scal}_g$ in M, where Scal_g is the Scalar curvature of the manifold. We show that these conditions are optimal by constructing examples of blowing-up solutions, with arbitrarily large energy, in the case of the round sphere with a constant potential function h.

GENERIC BLOW-UP RESULTS FOR LINEAR WAVES IN THE INTERIOR OF A SCHWARZSCHILD BLACK HOLE

Jan Sbierski
University of Oxford, UK
Joint work with **Grigorios Fournodavlos**.

I will discuss recent work [1], joint with Grigorios Fournodavlos, on the behaviour of generic solutions to the wave equation in the interior of a Schwarzschild black hole. We derive an asymptotic expansion of a general solution near the singularity at $r = 0$ and show that it is characterised by its first two leading order terms in r, a principal logarithmic term and a bounded second order term. Based on results [2], [3], [4] by Angelopoulos, Aretakis, and Gajic on the late time asymptotics of generic solutions to the wave equation in the exterior of a

Schwarzschild black hole we then show that the principal logarithmic term is non-vanishing in a neighbourhood of the asymptotic endpoints of the singular hypersurface $r = 0$.

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EXCITATION SPECTRUM OF TRAPPED BOSE-EINSTEIN CONDENSATES

Benjamin Schlein
Universität Zürich, Switzerland
Joint work with **C. Boccato, C. Brennecke, S. Cenatiempo, and S. Schraven**.

In this talk, we will discuss some recent results [1, 2, 3] concerning the ground state energy and the low-energy excitation spectrum of gases of N bosons trapped in a volume of order one and interacting through a repulsive potential with scattering length of the order $1/N$ (Gross-Pitaevskii regime). Our results confirm the validity of Bogoliubov theory.

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STABILITY OF EXPANDING BLACK HOLE COSMOLOGIES

Volker Schlue
University of Melbourne, Australia

In general relativity, an explicit family of solutions to the Einstein equations $\mathrm{Ric}(g) = \Lambda g$ with positive cosmological constant $\Lambda > 0$, the so-called *Kerr-de Sitter* space-times, describe the equilibrium states of a black hole in an expanding universe. The black hole exterior falls into two components, the stationary (near) zone, and the expanding (far) zone, separated by the cosmological horizon of the black hole. While the near region (bounded by the event and cosmological horizons) was recently proven to be dynamically stable [3], this talk reports on the dynamics of the far region (beyond the cosmological horizon) [5, 6]. Unlike in [3] (or the black hole stability problem for the *Kerr* solutions with $\Lambda = 0$ [1, 4]), the solution does not globally converge to an explicit family of solutions, but displays genuine asymptotic degrees of freedom; this was first observed for closed *de Sitter* cosmologies in [2]. Nonetheless we can prove that the conformal Weyl curvature decays [6] due to the expansion of the space-time geometry, which forms an essential part of the analysis of the cosmological region.

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THE DILUTE LIMIT OF INTERACTING BOSE GASES

Jan Philip Solovej
University of Copenhagen, Denmark
Joint work with **Birger Brietzke and Søren Fournais**.

I will discuss the interacting many-body Bose gase and, in particular, recent progress [2, 3] in understanding the asymptotics of the ground state energy in the dilute limit. The ground state energy of two bosons confined in a large box can be expressed in terms of the zero energy scattering length of the interacting potential. It has been a general belief in the physics literature [1, 4, 5] that the ground state energy in the dilute limit has a two term asymptotic expansion which is universal in the sense that the terms still depend only on the scattering length of the interaction potential. The asymptotics gives the celebrated Lee-Huang-Yang formula [4]. I will discuss recent progress of understanding this formula and the universality.

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RANDOM MATRIX MODELS AND TOPOLOGICAL RECURSIONS

Piotr Sułkowski
University of Warsaw, Poland & California Institute of Technology, USA

Within the past few decades we have witnessed great progress in the theory of random matrices. In particular, in the last one-and-a-half decade a powerful formalism of topological recursions - which can be interpreted as generalization of Ward identities for matrix models - has been developed. Topological recursions have already found a lot of applications in various branches of mathematics (in particular algebraic geometry, enumerative geometry, knot theory), high energy physics, statistical physics, and various other fields. In this talk I will explain what topological recursions are and summarize some of their applications in research areas mentioned above.

QUASINORMAL MODES OF BLACK HOLES

Claude Warnick
University of Cambridge, UK
Joint work with **Dejan Gajic**.

In recent years the problem of defining the quasinormal modes of subextremal black holes has been satisfactorily resolved for asymptotically de Sitter [1] and anti-de Sitter black holes [2]. The quasinormal frequencies may be realised as eigenvalues of a Fredholm operator resulting from a natural choice of coordinates on the black hole background. I will report on recent work with Dejan Gajic to extend this approach to treat extremal and asymptotically flat black hole spacetimes.

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LONG TIME DYNAMICS IN THE ROTATING EULER EQUATIONS

Klaus Widmayer
Ecole Polytechnique Fédérale de Lausanne, Switzerland
Joint work with **Yan Guo and Benoit Pausader**.

We investigate long time dynamics of solutions to the rotating Euler equations in three spatial dimensions. We develop a framework that is adapted to the symmetries and the dispersive properties of this problem and show how it can be used to understand the behavior of small data solutions, uniformly in the parameter of rotation.

The key idea is to use the available symmetries as much as possible, rather than to pursue a more brute force approach. While this streamlines the deduction of some energy type estimates, it also requires a fresh look at the (linear) dispersive estimates, deviating from the classical stationary phase intuition.