DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS SESSION D43

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D4

MULTISCALE METHODS AND ANALYSIS FOR THE DIRAC EQUATION IN THE NONRELATIVISTIC LIMIT REGIME

Weizhu Bao
National University of Singapore, Singapore

In this talk, I will review our recent works on numerical methods and analysis for solving the Dirac equation in the nonrelativistic limit regime, involving a small dimensionless parameter which is inversely proportional to the speed of light. In this regime, the solution is highly oscillating in time and the energy becomes unbounded and indefinite, which bring significant difficulty in analysis and heavy burden in numerical computation [4]. We begin with four frequently used finite difference time domain (FDTD) methods and the time splitting Fourier pseudospectral (TSFP) method and obtain their rigorous error estimates in the nonrelativistic limit regime by paying particularly attention to how error bounds depend explicitly on mesh size and time step as well as the small parameter [3]. Then we consider a numerical method by using spectral method for spatial derivatives combined with an exponential wave integrator (EWI) in the Gautschi-type for temporal derivatives to discretize the Dirac equation [3]. Rigorous error estimates show that the EWI spectral method has much better temporal resolution than the FDTD methods for the Dirac equation in the nonrelativistic limit regime [3]. We find that the time-splitting spectral method performs super-resolution in temporal discretization when the Dirac equation has no magnetic potential [5]. Based on a multiscale expansion of the solution, we present a multiscale time integrator Fourier pseudospectral (MTI-FP) method for the Dirac equation and establish its error bound which uniformly accurate in term of the small dimensionless parameter [1]. Numerical results demonstrate that our error estimates are sharp and optimal. Finally, these methods and results are then extended to the nonlinear Dirac

equation in the nonrelativistic limit regime [2]. This is a joint work with Yongyong Cai, Xiaowei Jia, Qinglin Tang and Jia Yin.

References

[1] W. Bao, Y. Cai, X. Jia and Q. Tang, *A uniformly accurate multiscale time integrator pseudospectral method for the Dirac equation in the nonrelativistic limit regime*, SIAM J. Numer. Anal. **54** (2016), 1785-1812.

[2] W. Bao, Y. Cai, X. Jia and J. Yin, *Error estimates of numerical methods for the nonlinear Dirac equation in the nonrelativistic limit regime*, Sci. China Math. **59** (2016), 1461-1494.

[3] W. Bao, Y. Cai, X. Jia and Q. Tang, *Numerical methods and comparison for the Dirac equation in the nonrelativistic limit regime*, J. Sci. Comput. **71** (2017), 1094-1134.

[4] W. Bao and J. Yin, *A fourth-order compact time-splitting Fourier pseudospectral method for the Dirac equation*, Res. Math. Sci. **6** (2019), article 11.

[5] W. Bao, Y. Cai and J. Yin, *Improved stability of optimal traffic paths*, Super-resolution of time-splitting methods for the Dirac equation in the nonrelativistic limit regime, arXiv: 1811.02174.

ACCELERATED SIMULATION FOR PLASMA KINETICS

Russel Caflisch

New York University, USA

Joint work with **Denis Silantyev and Bokai Yann**.

This presentation will discuss the kinetics of Coulomb collisions in plasmas, as described by the Landau-Fokker-Planck equation, and its numerical solution using a Direct Simulation Monte Carlo (DSMC) method. Acceleration of this method is achieved by coupling the particle method to a continuum fluid description. Efficiency of the resulting hybrid method is greatly increased by inclusion of particles with negative weights. This complicates the simulation, and introduces difficulties have plagued earlier efforts to use negatively weighted particles. This talk will describe significant progress that has been made in overcoming those difficulties.

OPTIMAL SAMPLING AND RECONSTRUCTION IN HIGH DIMENSION

Albert Cohen
Sorbonne Université, France

Motivated by non-intrusive approaches for high-dimensional parametric PDEs, we consider the general problem of approximating an unknown arbitrary function in any dimension from the data of point samples. The approximants are picked from given or adaptively chosen finite dimensional spaces. One principal objective is to obtain an approximation which performs as good as the best possible using a sampling budget that is linear in the dimension of the approximating space. We will show that this object if can is met by taking a random sample distributed according to a well chosen probability measure, and reconstructing by appropriate least-squares or pseudo-spectral methods.

MARGINAL TRIVIALITY OF THE SCALING LIMITS OF CRITICAL ISING AND φ^4 MODELS IN 4D

Hugo Duminil-Copin
IHÉS, France & University of Geneva, Switzerland
Joint work with **Michael Aizenman**.

The question of constructing a non-Gaussian field theory, i.e. a field with non-zero Ursell functions, is at the heart of Euclidean (quantum) field theory. While non-triviality results in $d < 4$ and triviality results in $d > 4$ were obtained in famous papers by Glimm, Jaffe, Aizenman, Frohlich and others, the crucial case of dimension 4 remained open. In this talk, we show that any continuum φ^4 theory constructed from Reflection Positive lattice φ^4 or Ising models is inevitably free in dimension 4. The proof is based on a delicate study of intersection properties of a non-Markovian random walk appearing in the random current representation of the model.

FROM WIGNER-DYSON TO PEARCEY: UNIVERSAL EIGENVALUE STATISTICS OF RANDOM MATRICES

László Erdős
Institute of Science and Technology, Austria

E. Wigner's revolutionary vision postulated that the local eigenvalue statistics of large random matrices are independent of the details of the matrix ensemble apart from its basic symmetry class. There have recently been a substantial development to prove Wigner's conjecture for larger and larger classes of matrix ensembles motivated by applications. They include matrices with entries with a general correlation structure and addition of deterministic matrices in a random relative basis. We also report on three types of universality, commonly known as the bulk, edge and cusp universality, referring to the behaviour of the density of states in the corresponding energy regime. While bulk and edge universalities have been subject to intensive research, the cusp universality has been studied only in very special cases before. Our recent work settles the question of this third and last type of universality in full generality.

DISSIPATIVE SOLUTIONS TO THE COMPRESSIBLE EULER SYSTEM

Eduard Feireisl
Czech Academy of Sciences, Czech Republic
Joint work with **Dominic Breit and Martina Hofmanová**.

We introduce the concept of (generalized) dissipative solutions to the compressible Euler system and review their basic properties:

• **Existence.** Dissipative solutions exist globally in time for any finite energy initial data.

• **Maximal dissipation, semigroup selection.** One can select a solution semigroup among dissipative solutions. The selected solution maximizes the energy dissipation (entropy production), see [1].

• **Weak-strong uniqueness.** A dissipative and a weak solution emanating from the same initial data coincide as soon as the weak solution belongs to certain Besov class and its velocity gradient satisfies a one sided Lipschitz condition, see [2].

• **Convergence of numerical schemes.** Cesaro avarages produced by suitable numerical schemes converge strongly to a dissipative solution, see [3].

References

[1] D. Breit, E. Feireisl, M. Hofmanová, *Solution semiflow to the isentropic Euler system*, Arxive Preprint Series, arXiv 1901.04798, 2019.

[2] E. Feireisl, S.S. Ghoshal, A. Jana, *On uniqueness of dissipative solutions to the isentropic Euler system*, Arxive Preprint Series, arXiv 1903.11687, 2019.

[3] E. Feireisl, M. Lukáčová–Medviďová, H. Mizerová, *$\mathcal{K}$-convergence as a new tool in numerical analysis*, Arxive Preprint Series, arXiv 1904.00297, 2019.

FINITE DIMENSIONAL STATE REPRESENTATION OF STRUCTURED POPULATION MODELS

Mats Gyllenberg
University of Helsinki, Finland

Structured population models can be formulated as delay systems. We consider the question of when delay systems, which are intrinsically infinite dimensional, can be represented by finite dimensional systems. Specifically, we give conditions for when all the information about the solutions of the delay system can be obtained from the solutions of a finite system of ordinary differential equations. For linear autonomous systems and linear systems with time-dependent input we give necessary and sufficient conditions and in the nonlinear case we give sufficient conditions. The ideas and results are illustrated by models for infectious diseases and physiologically structured populations.

QUANTITATIVE LINEAR STABILITY (HYPOCOERCIVITY) FOR CHARGED PARTICLES IN A CONFINING FIELD

Clément Mouhot
University of Cambridge, UK
Joint work with **K. Carrapatoso, J. Dolbeault, F. Hérau, S. Mischler, and C. Schmeiser**.

We report on recent joint results in which we develop quantitative methods for proving the existence of a spectral gap and estimating the gap, for hypocoercive kinetic equations that combine the local conservation laws of fluid mechanics and a confining potential force. The proofs involve a cascade of correctors and global commutator estimates, as well as new quantitative inequalities of Korn type. The latter extend to the case of the whole space with a potential force the classical Korn inequality in a bounded domain of elasticity theory. These results are a step towards constructing global solutions near equilibrium to the full nonlinear Boltzmann equation for charged particles subject to a confining potential.

STOCHASTIC MODELING AND OPTIMIZATION IN HUMAN-MACHINE INTERACTION SYSTEMS

Thaleia Zariphopoulou
University of Texas at Austin, USA
Joint work with **Agostino Capponi and Svein Olefsson**.

I will introduce a family of human-machine interaction (HMI) models in optimal asset allocation, risk management and portfolio choice (robo-advising). Modeling difficulties stem from the limited ability to quantify the human's risk preferences and describe their evolution, but also from the fact that the stochastic environment, in which the machine optimizes, itself adapts to real-time incoming information that is exogenous to the human. Furthermore, the human's risk preferences and the machine's states may evolve at different scales. This interaction creates an

adaptive cooperative game with asymmetric and incomplete information exchange between the two parties.

As a result, challenging questions arise on, among others, how frequently the two parties should communicate, what information can the machine accurately detect, infer and predict, how the human reacts to exogenous events and what are the effects on the machine's actions, how to improve the inter-linked reliability between the human and the machine, and others.

Such HMI models give rise to new, non-standard optimization problems that include well-posed and ill-posed sub-problems, and combine adaptive stochastic control, stochastic differential games, optimal stopping, multi-scales and learning.

References

[1] A. Capponi, S. Olefsson and T. Zariphopoulou, *Personalized robo-advising*, Preprint, 2019.

TALKS OF SESSION D43

PERFECT REPLICATION WITH MARKET IMPACT: TOWARDS A DUAL FORMULATION FOR A CLASS OF SECOND ORDER COUPLED FBSDEs

Bruno Bouchard
CEREMADE, Paris-Dauphine, PSL Research University, France
Joint work with **Grégoire Loeper, Halil Mete Soner, and Chao Zhou**.

We first extend the study of [2, 3] to stochastic target problems with general market impacts. The perfect hedging problem amounts to solving a second order coupled FBSDEs. Unlike [2, 3], the related fully non-linear PDE is not concave and the regularization/verification approach of [2] can not be applied. In place, we need to generalize the a priori estimates of [3] and exhibit smooth solutions from the classical parabolic equations theory. Up to an additional approximating argument, this allows us to show that the super-hedging price solves the parabolic equation and that a perfect hedging strategy can be constructed when the coefficients are smooth enough. This representation suggests a dual formulation for the Brownian diffusion Markovian setting. We shall explain how this dual formulation can indeed be exploited to solve a general class of non-Markovian second order coupled FBSDEs driven by general continuous martingales.

References

[1] B. Bouchard, G. Loeper, M. Soner and C. Zhou, *Second order stochastic target problems with generalized market impact*, arXiv:1806.08533, (2018).

[2] B. Bouchard, G. Loeper and Y. Zou, *Hedging of covered options with linear market impact and gamma constraint*, SIAM Journal on Control and Optimization **55(5)** (2017), 3319–3348.

[3] G. Loeper, *Option pricing with linear market impact and nonlinear Black-Scholes equations*, The Annals of Applied Probability **28(5)** (2018), 2664–2726.

OPTIMAL STRATEGIES IN A MARKET WITH MEMORY

Giulia Di Nunno
University of Oslo, Norway

We consider a market model driven by Volterra type dynamics driven by a time-changed Levy noise. These dynamics allow both for memory features and clustering effects in the trading times. In this framework, we study an optimal portfolio problem, which is then tackled via maximum principle. To produce such results we use different kind of information flows that take care of the time-change in adequate way and we rely on the non-anticipating stochastic derivative for random fields. Moreover, we study the solutions of Volterra type SDEs and Volterra type BSDEs driven by time-changed Levy noises.

MEAN-FIELD GAMES OF OPTIMAL STOPPING: A RELAXED SOLUTION APPROACH

Roxana Dumitrescu
King's College London, UK
Joint work with **Géraldine Bouveret and Peter Tankov**.

We consider the mean-field game where each agent determines the optimal time to exit the game by solving an optimal stopping problem with reward function depending on the density of the state processes of agents still present in the game. We place ourselves in the framework of relaxed optimal stopping, which amounts to looking for the optimal occupation measure of the stopper rather than the optimal stopping time. This framework allows us to prove the existence of the relaxed Nash equilibrium and the uniqueness of the associated value of the representative agent under mild assumptions. Further, we prove a rigorous relation between relaxed Nash equilibria and the notion of mixed solutions introduced in earlier works on the subject, and provide a criterion, under which the optimal strategies are pure strategies, that is, behave in a similar way to stopping times. Finally, we present a numerical method for computing the equilibrium in the case of potential games and show its convergence.

HILBERT SPACE-VALUED STOCHASTIC VOLATILITY MODELS AND AMBIT FIELDS

Heiðar Eyjólfsson
Reykjavík University, Iceland
Joint work with **Fred Espen Benth**.

We study Hilbert space-valued stochastic volatility models, and discuss representation and approximation of such processes. A typical application of such a model is the modelling of forward curves as an element in a given Hilbert space. Specifically, in a separable Hilbert space, a Lévy process driven variance process is introduced. We discuss ways of approximating the variance process in this setting. A problem of specific interest is how one obtains the square-root

of the variance process. We discuss ways of obtaining and approximating the square root in an infinite dimensional Hilbert space. We moreover relate these models to the class Hilbert space-valued volatility modulated Volterra processes we call Hambit fiels [2]. Hambit fields are Hilbert space-valued analogues of ambit fields as introduced by Barndorff-Nielsen and Schmiegel [1]. Hambit fields can be expressed as a countable sum of weighted real-valued volatility modulated Volterra processes, for which Ornstein-Uhlenbeck process constitute a particular case. Hambit fields can moreover be interpreted as the boundary of the mild solution of a certain first order stochastic partial differential equation.

References

[1] O.E. Barndorff-Nielsen and J. Schmiegel, *Lévy-based tempo-spatial modelling; with applications to turbulence*, Uspekhi Mat. NAUK **59** (2004), 65-91.

[2] F.E. Benth and H. Eyjolfsson, *Representation and approximation of ambit fields in Hilbert space*, Stochastics **89** (2017), 311-347

3R HYBRID SCHEME FOR BROWNIAN SEMISTATIONARY PROCESSES

Masaaki Fukasawa
Osaka University, Japan
Joint work with **Asuto Hirano**.

The Brownian semistationary process has attracted much attention recently in quantitative finance in the context of rough volatility modeling. We propose a new numerical approximation scheme, 3R hybrid scheme, which refines the hybrid scheme proposed by Bennedsen et al. [1] for Brownian semistationary processes. The mean squared error is shown to be significantly reduced while computational costs remain almost the same. The key idea is to reuse random variables through orthogonal projections.

References

[1] M. Bennedsen, A. Lunde and M.S. Pakkanen, *Hybrid scheme for Brownian semistationary processes*, Finance and Stochastics **21** (2017), 931-965.

A MEAN-FIELD GAME PRICE MODEL

Diogo Gomes
KAUST, Saudi Arabia
Joint work with **João Saúde**.

Here, we introduce a price-formation model where a large number of small players can store and trade electricity. Our model is a constrained mean-field game (MFG) where the price is a Lagrange multiplier for the supply vs. demand balance condition. We establish the existence of a unique solution using a fixed-point argument. In particular, we show that the price is well-defined and it is a Lipschitz function of time. Then, we study linear-quadratic models that can be solved explicitly and compare our model with real data.

IT'S ALL RELATIVE: MEAN FIELD GAME EXTENSIONS OF MERTON'S PROBLEM

Olivier Guéant
Université Paris 1 Panthéon-Sorbonne, France
Joint work with **Alexis Bismuth**.

Merton's problem deals with the optimal investment and consumption choices of economic agents. The classical results of Merton have been extended to add many features, but never,

as far as we know, to take account of jealousy. In this talk, we show how the introduction of jealousy modifies Merton's problem and results in a problem of the mean field game (MFG) type (a mean field game of controls in fact). Interestingly, many analytical results can be obtained and will be presented, along with applications.

FROM APPLICATIONS TO EQUATIONS

Piotr Jaworski
University of Warsaw, Poland

In mathematical finance, the standard approach is to model the logarithms of prices of financial assets as Wiener processes, which are correlated in a deterministic way. In my talk I will deal with a pair of Wiener processes W^1 and W^2 which are randomly correlated. Under the assumption that the quadratic covariation of W^1 and W^2 can be described by a *deterministic* function of W^1 and W^2,

$$d\langle W^1, W^2\rangle_t = A(t, W^1_t, W^2_t)dt,$$

I will show that the joint distribution function $F(t, x_1, x_2)$ and the copula $C(t, u, v)$ of the pair (W^1, W^2) are generalized weak solutions of parabolic partial differential equations.

STOCHASTIC CONTROL OF MEASURE-VALUED MARTINGALES WITH APPLICATIONS TO ROBUST FINANCE

Sigrid Källblad
Technische Universität Wien, Austria
Joint work with **A. Cox, M. Larsson, and S. Svaluto.**.

Motivated by robust pricing problems in mathematical finance, we consider in this talk a specific

constrained optimisation problem. Our approach is based on reformulating this problem as an optimisation problem over so-called measure-valued martingales (MVMs) enabling the problem to be addressed by use of dynamic programming methods. In the emerging stochastic control problem MVMs appear as weak solutions to a specific SDE for which we prove existence of solutions; we then show that our control problem satisfies the Dynamic Programming Principle and relate the value function to a certain HJB-type equation. A key motivation for the study of control problems featuring MVMs is that a number of interesting probabilistic problems can be formulated as such optimisation problems; we illustrate this by applying our results to optimal Skorokhod embedding problems as well as robust pricing problems.

THE EXPRESSIVENESS OF RANDOM DYNAMICAL SYSTEMS

Martin Larsson
Carnegie Mellon University, USA
Joint work with **Christa Cuchiero and Josef Teichmann**.

Deep neural networks perform exceedingly well on a variety of learning tasks, in particular in finance where they are quickly gaining importance. Training a deep neural network amounts to optimizing a nonlinear objective over a very large space of parameters. This would seem a hopeless task if a globally optimal solution were required. The fact that this can succeed suggests that the result is largely insensitive to the detailed structure of the selected locally near-optimal solution, a perspective that is supported by empirical evidence. In this work we attempt a step toward a theoretical understanding of this phenomenon. In a model of deep neural networks as discretizations of controlled dynamical systems, we rigorously prove that any learning task can be accomplished even if a majority of the parameters are chosen at random

DUALITY FOR RISK FUNCTIONALS ON ORLICZ SPACES

Cosimo Munari
Universität Zürich, Switzerland
Joint work with **Niushan Gao, Denny Leung, Foivos Xanthos**.

A well-known result by Delbaen states that every convex risk measure defined on the space of bounded positions is automatically weak-star lower semicontinuous whenever it satisfies the Fatou property. This allows to derive a nice dual representation where the constraint set of dual elements consists of countably-additive measures. This result is no longer true if one abandons the bounded setting. The objective of the talk is to show that a dual representation with countably-additive measures holds in a general Orlicz space if the risk measure is additionally assumed to be either law invariant (the risk measure depends only on the probability law of a risky position) or surplus invariant (the risk measure depends, in a suitable way, only on the downside of a risky position).

References

[1] N. Gao, D. Leung, C. Munari, F. Xanthos, *Fatou property, representations, and extensions of law-invariant risk measures on general Orlicz spaces*, Finance and Stochastics **22** (2018), 395–415.

[2] N. Gao, C. Munari, *Surplus-invariant risk measures*, Mathematics of Operations Research, to appear.

ON STOCHASTIC LANGEVIN AND FOKKER-PLANCK EQUATIONS

Andrea Pascucci
University of Bologna, Italy
Joint work with **Antonello Pesce**.

We study existence, regularity in Hölder classes and estimates from above and below of the fundamental solution of a degenerate SPDE satisfying the weak Hörmander condition. Our method is based on a Wentzell's reduction of the SPDE to a PDE with random coeffcients to which we apply the parametrix technique to construct a fundamental solution. This approach avoids the use of the Duhamel's principle for the SPDE and the related measurability issues that appear in the stochastic framework. Applications to stochastic filtering are also discussed.

References

[1] N.V. Krylov, *Hörmander's theorem for stochastic partial differential equations*, Algebra i Analiz **27(3)** (2015), 157-182.

[2] N.V. Krylov and A. Zatezalo, *A direct approach to deriving filtering equations for diffusion processes*, Appl. Math. Optim. **42(3)** (2000), 315-332.

[3] H. Kunita, *Stochastic flows and stochastic differential equations*, Vol. 24 of Cambridge Studies in Advanced Mathematics, Cambridge University Press, Cambridge, 1990.

[4] E. Lanconelli and S. Polidoro, *On a class of hypoelliptic evolution operators*, Rend. Sem. Mat. Univ. Politec. Torino **52(1)** (1994), 29-63.

[5] P.L. Lions, *On Boltzmann and Landau equations*, Philos. Trans. Roy. Soc. London Ser. A 346 **1679** (1994), 191-204.

[6] A. Pascucci and A. Pesce, *The parametrix method for parabolic SPDEs*, (2019), arXiv:1803.06543v3.

RISK SENSITIVE DYADIC IMPULSE CONTROL FOR UNBOUNDED PROCESSES

Marcin Pitera
Jagiellonian University in Kraków, Poland
Joint work with **Łukasz Stettner**.

Dyadic impulse control of continuous time Feller-Markov processes with risk-sensitive long-run average cost is considered. The uncontrolled process is assumed to be bounded in the weighted norm and to be ergodic; the process could be unbounded in the supremum norm and do not necessarily satisfy uniform ergodicity property. The existence of solution to suitable Bellman

equation using local span contraction method is shown, and link to optimal problem solution is established with the help of Hölder's (entropic) inequalities.

A GENERAL APPROACH TO NON–MARKOVIAN TIME-INCONSISTENT STOCHASTIC CONTROL FOR SOPHISTICATED PLAYERS

Dylan Possamaï
Columbia University, USA
Joint work with **Camilo Hernández**.

This paper is the first attempt at a general non-Markovian theory of time–inconsistent stochastic control problems in continuous-time. We consider sophisticated agents who are aware of their time-inconsistency and take into account in future decisions. We prove here that equilibria in such a problem can be characterised through a new type of multi-dimensional system of backward SDEs, for which we obtain wellposedness. Unlike the existing literature, we can treat the case of non-Markovian dynamics, and our results go beyond verification type theorems, in the sense that we prove that any (strict) equilibrium must necessarily arise from our system of BSDEs. This is a joint work with Camilo Hernández, Columbia University.

MARTINGALE OPTIMAL TRANSPORT DUALITY AND ROBUST FINANCE

David Prömel
University of Oxford, UK
Joint work with **Patrick Cheridito, Matti Kiiski, and H. Mete Soner**.

Without assuming any probabilistic price dynamics, we consider a frictionless financial market given by the Skorokhod space, on which some financial options are liquidly traded. In this model-free setting we show various pricing-hedging dualities and the analogue of the fundamental theorem of asset pricing. For this purpose we study the corresponding martingale optimal transport (MOT) problem: We obtain a dual representation of the Kantorovich functional (super-replication functional) defined for functions (financial derivatives) on the Skorokhod space using quotient sets (hedging sets). Our representation takes the form of a Choquet capacity generated by martingale measures satisfying additional constraints to ensure compatibility with the quotient sets. As an immediate consequence of the duality result, we deduce a general robust fundamental theorem of asset pricing.

References

[1] P. Cheridito, M. Kiiski, D. J. Prömel, H. M. Soner, *Martingale Optimal Transport Duality*, ArXiv Preprint arXiv:1904.04644.

STOCHASTIC VOLTERRA EQUATIONS

Sergio Pulido
ENSIIE & Université Paris-Saclay, France
Joint work with **Eduardo Abi Jaber, Christa Cuchiero, and Martin Larsson**.

We obtain general weak existence and stability results for Stochastic Convolution Equations (SVEs) with jumps under mild regularity assumptions, allowing for non-Lipschitz coefficients and singular kernels. The motivation to study SVEs comes from the literature on rough volatility models. Our approach relies on weak convergence in L^p spaces. The main tools are new a priori estimates on Sobolev-Slobodeckij norms of the solution, as well as a novel martingale problem that is equivalent to the original equation. This leads to generic approximation and stability theorems in the spirit of classical martingale problem theory. To illustrate the applicability of our results, we consider scaling limits of nonlinear Hawkes processes and approximations of stochastic Volterra processes by Markovian semimartingales.

RISK MEASURES AND PROGRESSIVE ENLARGEMENT OF FILTRATIONS: A BSDE APPROACH

Emanuela Rosazza Gianin
University of Milano-Bicocca, Italy
Joint work with **Alessandro Calvia**.

The aim of the talk is to investigate dynamic risk measures in the case of enlargement of filtration and its impact on the corresponding risk measure and on its properties. More precisely, we show how to induce a dynamic risk measure from a BSDE whose noise is given by a Brownian motion and a marked point process. In terms of the underlying information flow, this corresponds to a progressive enlargement of a Brownian filtration with information brought by the occurrence of random events at random times. This may describe the presence of defaults. The class of BSDEs with jumps considered was introduced in [1]. In the single jump case, we show that dynamic risk measures induced by these BSDEs admit a decomposition into two risk measures, one before and the other after the default. Furthermore, we prove that standard properties of dynamic risk measures are guaranteed by similar properties of the driver of these BSDEs and that time-consistency holds. From a financial point of view, the decomposition of the "global" risk measure into different "local" ones is reasonable. Before and after a default time, indeed, the risk measure should be updated in order to take into account the new information.

References

[1] I. Kharroubi, T. Lim, *Progressive enlargement of filtrations and backward stochastic differential equations with jumps*, J. Theoret. Probab. **27** (2014), 683-724.

ASYMPTOTIC SYNTHESIS OF CONTINGENT CLAIMS IN A SEQUENCE OF DISCRETE-TIME MARKETS

Walter Schachermayer
Universität Wien, Austria
Joint work with **M. Kreps**.

We prove a connection between discrete-time models of financial markets and the celebrated Black-Scholes-Merton continuous-time model in which "markets are complete." Specifically, we prove that if (a) the probability law of a sequence of discrete-time models converges (in the functional sense) to the probability law of the Black-Scholes-Merton model, and (b) the largest possible one-period step in the discrete-time models converges to zero, then every bounded and continuous contingent claim can be asymptotically synthesized with bounded risk: For any $\epsilon > 0$, a consumer in the discrete-time economy far enough out in the sequence can synthesize a claim that is no more than ϵ different from the target contingent claim x with probability at least $1 - \epsilon$, and which, with probability 1, has norm less or equal to the norm of the target claim. This shows that, in terms of important economic properties, the Black-Scholes-Merton model, with its complete markets, idealizes many more discrete-time models than models based on binomial random walks.

PORTFOLIO OPTIMIZATION WITH TRANSACTION COSTS - DIRECT APPROACH

Łukasz Stettner
Polish Academy of Sciences, Poland
Joint work with **Tomasz Rogala**.

We consider discrete time market with general bid and ask prices and will be interested in maximization of utility from terminal wealth. Our approach will be direct, we shall not use well developed duality theory (see for example [1] and references therein). The result is based

on an analysis of general dynamic programming equation and work with suitable selectors. We show in particular the existence and form of shadow price i.e. the price on the market without proportional transaction costs for which the strategies and the value of the functional (utility from terminal wealth) are the same as in the case of market with transaction costs. Such results are obtained under general conditions on the utility function as well as on the bid and ask prices, both in one as well as in multidimensional case. Results presented are based on the papers [2], [3] and [4].

References

[1] C. Czichowski, J. Muhle-Karbe, W. Schachermayer, *ransaction costs, shadow prices, and duality in discrete time*, SIAM J. Financial Math. **5** (2014), 258–277.

[2] T. Rogala, Ł. Stettner, *On Construction of Discrete Time Shadow Price*, Appl. Math. Optim. **72(3)** (2015), 391-433.

[3] T. Rogala, Ł. Stettner, *Optimal strategies for utility from terminal wealth with general bid and ask prices*, Appl. Math. Optim. (2019) published online.

[4] T. Rogala, Ł. Stettner, *Multidimensional shadow price with general bid and ask prices*, in preparation.

DUALITY FOR HOMOGENEOUS OPTIMISATION PROBLEMS

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Joint work with **David Driver**.

This talk is concerned with stochastic optimal control problems with a certain homogeneity. For such problems, a novel dual problem is formulated. The results are applied to a stochastic volatility variant of the classical Merton problem. Another application of this duality is to the study the right-most particle of a branching Levy process.