DYNAMICS, EQUATIONS AND APPLICATIONS

BOOK OF ABSTRACTS
SESSION D44

AGH University of Science and Technology

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PLENARY LECTURES

GENERIC CONSERVATIVE DYNAMICS

Artur Avila
Universität Zürich, Switzerland & IMPA, Brazil

ON THE REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC PDES

Alessio Figalli
ETH Zürich, Switzerland

Stable solutions to semilinear elliptic PDEs appear in several problems. It is known since the 1970's that, in dimension $n > 9$, there exist singular stable solutions. In this talk I will describe a recent work with Cabré, Ros-Oton, and Serra, where we prove that stable solutions in dimension $n \leq 9$ are smooth. This answers also a famous open problem, posed by Brezis, concerning the regularity of extremal solutions to the Gelfand problem.

RANDOM LOOPS

Martin Hairer
Imperial College London, UK

2D PERCOLATION REVISITED

Stanislav Smirnov
University of Geneva, Switzerland & Skoltech, Russia
Joint work with **Mikhail Khristoforov**.

We will discuss the state of our understanding of 2D percolation, and will present a recent joint work with Mikhail Khristoforov, giving a new proof of its conformal invariance at criticality.

STABILITY AND NONLINEAR PDES IN MIRROR SYMMETRY

Shing-Tung Yau
Harvard University, USA

I shall give a talk about a joint work that I did with Tristan Collins on an important nonlinear system equation of Monge-Ampère type. It is motivated from the theory of Mirror symmetry in string theory. I shall also talk about its algebraic geometric meaning.

FROM CLASSICAL TO QUANTUM AND BACK

Maciej Zworski
University of California, Berkeley, USA

Microlocal analysis exploits mathematical manifestations of the classical/quantum (particle/wave) correspondence and has been a successful tool in spectral theory and partial differential equations. We can say that these two fields lie on the "quantum/wave side".

In the last few years microlocal methods have been applied to the study of classical dynamical problems, in particular of chaotic flows. That followed the introduction of specially tailored spaces by Blank-Keller-Liverani, Baladi-Tsujii and other dynamicists and their microlocal interpretation by Faure-Sjoestrand and by Dyatlov and the speaker.

I will explain this microcar/dynamical connection in the context of Ruelle resonances, decay of correlations and meromorphy of dynamical zeta functions. I will also present some recent advances, among them results by Dyatlov-Guillarmou (Smale's conjecture on meromorphy of zeta functions for Axiom A flows), Guillarmou-Lefeuvres (local determination of metrics by the length spectrum) and Dang-Rivière (Ruelle resonances and Witten Laplacian).

PUBLIC LECTURE

FROM OPTIMAL TRANSPORT TO SOAP BUBBLES AND CLOUDS: A PERSONAL JOURNEY

Alessio Figalli
ETH Zürich, Switzerland

In this talk I'll give a general overview, accessible also to non-specialists, of the optimal transport problem. Then I'll show some applications of this theory to soap bubbles (isoperimetric inequalities) and clouds (semigeostrophic equations), problems on which I worked over the last 10 years. Finally, I will conclude with a brief description of some results that I recently obtained on the study of ice melting into water.

INVITED TALKS OF PART D4

MULTISCALE METHODS AND ANALYSIS FOR THE DIRAC EQUATION IN THE NONRELATIVISTIC LIMIT REGIME

Weizhu Bao
National University of Singapore, Singapore

In this talk, I will review our recent works on numerical methods and analysis for solving the Dirac equation in the nonrelativistic limit regime, involving a small dimensionless parameter which is inversely proportional to the speed of light. In this regime, the solution is highly oscillating in time and the energy becomes unbounded and indefinite, which bring significant difficulty in analysis and heavy burden in numerical computation [4]. We begin with four frequently used finite difference time domain (FDTD) methods and the time splitting Fourier pseudospectral (TSFP) method and obtain their rigorous error estimates in the nonrelativistic limit regime by paying particularly attention to how error bounds depend explicitly on mesh size and time step as well as the small parameter [3]. Then we consider a numerical method by using spectral method for spatial derivatives combined with an exponential wave integrator (EWI) in the Gautschi-type for temporal derivatives to discretize the Dirac equation [3]. Rigorous error estimates show that the EWI spectral method has much better temporal resolution than the FDTD methods for the Dirac equation in the nonrelativistic limit regime [3]. We find that the time-splitting spectral method performs super-resolution in temporal discretization when the Dirac equation has no magnetic potential [5]. Based on a multiscale expansion of the solution, we present a multiscale time integrator Fourier pseudospectral (MTI-FP) method for the Dirac equation and establish its error bound which uniformly accurate in term of the small dimensionless parameter [1]. Numerical results demonstrate that our error estimates are sharp and optimal. Finally, these methods and results are then extended to the nonlinear Dirac equation in the nonrelativistic limit regime [2]. This is a joint work with Yongyong Cai, Xiaowei Jia, Qinglin Tang and Jia Yin.

References

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ACCELERATED SIMULATION FOR PLASMA KINETICS

Russel Caflisch
New York University, USA
Joint work with **Denis Silantyev and Bokai Yann**.

This presentation will discuss the kinetics of Coulomb collisions in plasmas, as described by the Landau-Fokker-Planck equation, and its numerical solution using a Direct Simulation Monte Carlo (DSMC) method. Acceleration of this method is achieved by coupling the particle method to a continuum fluid description. Efficiency of the resulting hybrid method is greatly increased by inclusion of particles with negative weights. This complicates the simulation, and introduces difficulties have plagued earlier efforts to use negatively weighted particles. This talk will describe significant progress that has been made in overcoming those difficulties.

OPTIMAL SAMPLING AND RECONSTRUCTION IN HIGH DIMENSION

Albert Cohen
Sorbonne Université, France

Motivated by non-intrusive approaches for high-dimensional parametric PDEs, we consider the general problem of approximating an unknown arbitrary function in any dimension from the data of point samples. The approximants are picked from given or adaptively chosen finite dimensional spaces. One principal objective is to obtain an approximation which performs as good as the best possible using a sampling budget that is linear in the dimension of the approximating space. We will show that this object if can is met by taking a random sample distributed according to a well chosen probability measure, and reconstructing by appropriate least-squares or pseudo-spectral methods.

MARGINAL TRIVIALITY OF THE SCALING LIMITS OF CRITICAL ISING AND φ^4 MODELS IN 4D

Hugo Duminil-Copin
IHÉS, France & University of Geneva, Switzerland
Joint work with **Michael Aizenman**.

The question of constructing a non-Gaussian field theory, i.e. a field with non-zero Ursell functions, is at the heart of Euclidean (quantum) field theory. While non-triviality results in $d < 4$ and triviality results in $d > 4$ were obtained in famous papers by Glimm, Jaffe, Aizenman, Frohlich and others, the crucial case of dimension 4 remained open. In this talk, we show that any continuum φ^4 theory constructed from Reflection Positive lattice φ^4 or Ising models is inevitably free in dimension 4. The proof is based on a delicate study of intersection properties of a non-Markovian random walk appearing in the random current representation of the model.

FROM WIGNER-DYSON TO PEARCEY: UNIVERSAL EIGENVALUE STATISTICS OF RANDOM MATRICES

László Erdős
Institute of Science and Technology, Austria

E. Wigner's revolutionary vision postulated that the local eigenvalue statistics of large random matrices are independent of the details of the matrix ensemble apart from its basic symmetry class. There have recently been a substantial development to prove Wigner's conjecture for larger and larger classes of matrix ensembles motivated by applications. They include matrices with entries with a general correlation structure and addition of deterministic matrices in a random relative basis. We also report on three types of universality, commonly known as the bulk, edge and cusp universality, referring to the behaviour of the density of states in the corresponding energy regime. While bulk and edge universalities have been subject to intensive research, the cusp universality has been studied only in very special cases before. Our recent work settles the question of this third and last type of universality in full generality.

DISSIPATIVE SOLUTIONS TO THE COMPRESSIBLE EULER SYSTEM

Eduard Feireisl
Czech Academy of Sciences, Czech Republic
Joint work with **Dominic Breit and Martina Hofmanová**.

We introduce the concept of (generalized) dissipative solutions to the compressible Euler system and review their basic properties:

• **Existence.** Dissipative solutions exist globally in time for any finite energy initial data.

• **Maximal dissipation, semigroup selection.** One can select a solution semigroup among dissipative solutions. The selected solution maximizes the energy dissipation (entropy production), see [1].

• **Weak-strong uniqueness.** A dissipative and a weak solution emanating from the same initial data coincide as soon as the weak solution belongs to certain Besov class and its velocity gradient satisfies a one sided Lipschitz condition, see [2].

• **Convergence of numerical schemes.** Cesaro avarages produced by suitable numerical schemes converge strongly to a dissipative solution, see [3].

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FINITE DIMENSIONAL STATE REPRESENTATION OF STRUCTURED POPULATION MODELS

Mats Gyllenberg
University of Helsinki, Finland

Structured population models can be formulated as delay systems. We consider the question of when delay systems, which are intrinsically infinite dimensional, can be represented by finite dimensional systems. Specifically, we give conditions for when all the information about the solutions of the delay system can be obtained from the solutions of a finite system of ordinary differential equations. For linear autonomous systems and linear systems with time-dependent input we give necessary and sufficient conditions and in the nonlinear case we give sufficient conditions. The ideas and results are illustrated by models for infectious diseases and physiologically structured populations.

QUANTITATIVE LINEAR STABILITY (HYPOCOERCIVITY) FOR CHARGED PARTICLES IN A CONFINING FIELD

Clément Mouhot
University of Cambridge, UK
Joint work with **K. Carrapatoso, J. Dolbeault, F. Hérau, S. Mischler, and C. Schmeiser**.

We report on recent joint results in which we develop quantitative methods for proving the existence of a spectral gap and estimating the gap, for hypocoercive kinetic equations that combine the local conservation laws of fluid mechanics and a confining potential force. The proofs involve a cascade of correctors and global commutator estimates, as well as new quantitative inequalities of Korn type. The latter extend to the case of the whole space with a potential force the classical Korn inequality in a bounded domain of elasticity theory. These results are a step towards constructing global solutions near equilibrium to the full nonlinear Boltzmann equation for charged particles subject to a confining potential.

STOCHASTIC MODELING AND OPTIMIZATION IN HUMAN-MACHINE INTERACTION SYSTEMS

Thaleia Zariphopoulou
University of Texas at Austin, USA
Joint work with **Agostino Capponi and Svein Olefsson**.

I will introduce a family of human-machine interaction (HMI) models in optimal asset allocation, risk management and portfolio choice (robo-advising). Modeling difficulties stem from the limited ability to quantify the human's risk preferences and describe their evolution, but also from the fact that the stochastic environment, in which the machine optimizes, itself adapts to real-time incoming information that is exogenous to the human. Furthermore, the human's risk preferences and the machine's states may evolve at different scales. This interaction creates an

adaptive cooperative game with asymmetric and incomplete information exchange between the two parties.

As a result, challenging questions arise on, among others, how frequently the two parties should communicate, what information can the machine accurately detect, infer and predict, how the human reacts to exogenous events and what are the effects on the machine's actions, how to improve the inter-linked reliability between the human and the machine, and others.

Such HMI models give rise to new, non-standard optimization problems that include well-posed and ill-posed sub-problems, and combine adaptive stochastic control, stochastic differential games, optimal stopping, multi-scales and learning.

References

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TALKS OF SESSION D44

INFINITE-DIMENSIONAL INVERSE PROBLEMS WITH FINITE MEASUREMENTS

Giovanni S. Alberti
University of Genoa, Italy
Joint work with **Matteo Santacesaria**.

In this talk I will discuss how ideas from applied harmonic analysis, in particular sampling theory and compressed sensing, may be applied to inverse problems for partial differential equations. The focus will be on inverse boundary value problems for the conductivity and the Schrodinger equations, but the approach is very general and allows to handle many other classes of inverse problems. I will give uniqueness and stability results, both in the linearized and in the nonlinear case. These results make use of a recent general theory of infinite-dimensional compressed sensing for deterministic and non-isometric operators, which will be briefly surveyed.

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RATE INDEPENDENT EVOLUTIONS: DIFFERENTIAL SENSITIVITY

Martin Brokate
Technische Universität München, Germany

Rate independent evolutions are inherently nonsmooth. Mappings involving rate independent evolutions do not possess classical derivatives. However, basic rate independent processes like the scalar play or stop hysteresis operator have weak derivatives in the sense of Bouligand. Moreover, they are semismooth. As a consequence, the control-to-state mapping of certain evolutions which include rate independent elements also enjoy these properties. This work is published in [1, 2], based on the previous work [3].

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SOLVING HYBRID INVERSE PROBLEMS MOST OF THE TIME

Yves Capdeboscq
Université de Paris, France
Joint work with **Giovanni S. Alberti**.

We show that combining a projection method attributed to H. Whitney in geometry, and a unique continuation principle for solution of 2nd order PDEs, a small number of boundary condition can be selected so that the vector space generated by the gradient fields of electrical potentials be of maximal rank everywhere in the domain. This result applies in particular when the conductivity of the medium varies in space. This provides a generalization of the so-called thermal coordinates beyond the two dimensional case.

INTEGRAL REPRESENTATION OF LOCAL ENERGIES ON BD

Marco Caroccia
University of Florence, Italy
Joint work with **Matteo Focardi and Nicolas Van Goethem**.

In collaboration with Matteo Focardi and Nicolas Van Goethem we make use of the result on the structure of the $\mathcal{A}$-free measure provided in [1] to give a full integral representation result for local energies defined on the space of bounded deformation maps $BD(\Omega)$ which accounts also for the Cantor part of the symmetric gradient measure. We employ the *global method* introduced in [2] and exploited also in [3] to give an integral representation Theorem for energies defined on $SBD(\Omega)$. A full integral representation result for energy on $BD(\Omega)$ has been missing until now due to a lack of information on the structure of the Cantor part of the symmetric gradient measure. This piece of information has been finally provided by Rindler, De Philippis in the celebrated result of 2016 [1] and it represents the crucial ingredient of our proof, together with a double blow-up procedure and a fine analysis of the blow-up limits around Cantor points. I will briefly describe the methodology of the proof and the consequences of our result in fracture mechanics and in damage models.

References

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UNRESOLVED SCALES AND STOCHASTIC PARAMETERIZATION IN ATMOSPHERE-OCEAN MODELING

Daan Crommelin
CWI & University of Amsterdam, Netherlands

The question how to represent or parameterize processes at unresolved scales remains an important issue in atmosphere-ocean modeling, posing both practical and theoretical challenges. For multiscale dynamical systems such as atmosphere, ocean and climate, the theoretical framework of the Mori-Zwanzig (MZ) formalism can help to guide the development of reduced (or coarse-grained) models in which processes at small/fast scales are no longer explicitly resolved but instead parametrized by stochastic models terms [1]. In practice it is often not feasible to derive these reduced models including their stochastic closure analytically following MZ. However, the formalism gives insight in suitable functional forms of the reduced models and in particular in the role of memory terms. This insight can be useful in data-based approaches, where model closures or parameterizations are extracted from available data. I will discuss work where such a data-based approach is developed and used in atmosphere-ocean modeling. As part of this approach, both resampling methods [2, 4] and discrete models [3] have been explored.

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ACCURACY OF NONLINEAR FILTERS

Jana de Wiljes
Universität Potsdam, Germany
Joint work with **Xin Tong**.

In the context of nonlinear high dimensional filtering problems ensemble based techniques such as the Ensemble Kalman Filter are still consider state of the art despite the lack of mathematical foundation in this setting. In a recent study long time stability and accuracy results for the deterministic Ensemble Kalman Bucy Filter were derived for a setting with fully observed state subject to small measurement noise [1]. Here these results are extended to the case where the ensemble size is larger than the state space for a localized deterministic Ensemble Kalman Bucy Filter. In contrast to the previously derived bounds the bounds for the localized filter are independent of the ensemble size. Further a dimension independent bound is obtained for the individual components of the error and a Laplace type condition holds.

References

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MATHEMATICAL ANALYSIS OF EXCHANGE DRIVEN GROWTH: FUNDAMENTALS AND LARGE TIME BEHAVIOUR

Emre Esenturk
University of Warwick, UK
Joint work with **Juan Velazquez**.

Exchange-driven growth is a process (represented by an infinite set of ODEs) in which pairs of clusters interact and exchange single unit of mass at a time. In the recent years EDG has been used to model several natural and social phenomena The rate of exchange is given by an interaction kernel $K(j,k)$ which depends on the masses of the two interacting clusters (of sizes j and k). Despite its wide use first mathematical analyses of this system were provided only recently [1].

In this talk we present results on the fundamentals properties (existence, uniqueness, nonexistence and etc) and the large time behaviour. For the existence, we show two different sets of results depending on whether $K(j,k)$ is symmetric or not. In the case of non-symmetric kernels we present global existence and uniqueness of solutions for kernels satisfying $K(j,k) \leq Cjk$. This result is optimal in the sense that for faster growing kernels the solutions cannot exist (up to some technical assumptions). On the other hand, in the case of symmetric kernels we show that global unique solutions exist for kernels satisfying $K(j,k) \leq j^{\mu}k^{\nu} + j^{\nu}k^{\mu}$ ($\mu+\nu \leq 3$ and $\mu,\nu \leq 2$) and that the nonexistence is also "delayed". For the large time behavior, we again show two sets of result for separable type kernels. Under some technical assumptions, we show that the system admits equilibrium solutions up to a critical mass above which there is no equilibrium. We show that if the system has an initial mass above the critical mass then the solutions converge to critical equilibrium distribution in a weak sense while strong convergence can be shown when initial mass is below.

References

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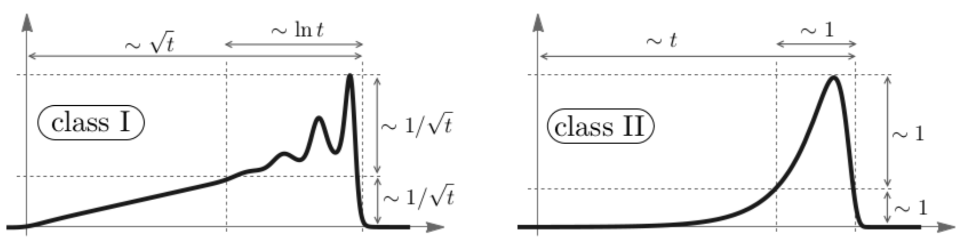
LONGTIME BEHAVIOR AND ASYMPTOTIC REGIMES FOR SMOLUCHOWSKI EQUATIONS

Michael Herrmann
Technische Universität Braunschweig, Germany
Joint work with **Barbara Niethammer and Juan J.L. Velázquez**.

Smoluchowski's coagulation equation is the most fundamental dynamical model for mass aggregation and appears in many different branches of physics, chemistry, biology, and materials science. The mathematical properties of this nonlinear integral equation, however, are only partially understood and depend intimately on the chosen coagulation kernel.

In this talk we discuss several asymptotic regimes for kernel functions and investigate the longtime behavior of solutions by combining asymptotic analysis, heuristic arguments, and numerical simulations. In particular, we study traveling waves and self-similar profiles for near-diagonal kernels with homogeneity one and provide analytical or numerical evidence for the onset of instabilities and the formation of oscillations. We further sketch the challenges in the numerical computation of self-similar solutions and initial value problems.



Cartoon of the self-similar solutions in two different asymptotic regimes.

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MATHEMATICS FOR STEEL PRODUCTION AND MANUFACTURING

Dietmar Hömberg
WIAS & Technische Universität Berlin, Germany
Joint work with **Manuel Arenas, Prerana Das, and Robert Lasarzik**.

In my presentation I will discuss some results from the European Industrial Doctorate project "MIMESIS - Mathematics and Materials Science for steel production and manufacturing". Applications cover tube welding, induction hardening and flame cutting.

The mathematical models considered combine a vector potential formulation of Maxwell's equations with a nonlinear heat equation and an evolution equation for the change of microstructure. In the presentation we analyse the well-posedness of these multi-field problems, discuss related optimal control problems and show some simulation results related to real industrial use cases.

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PROGRESS IN THE ANALYSIS OF DOMAIN WALLS IN THIN FERROMAGNETIC FILMS

Radu Ignat
Université Paul Sabatier, France

Ferromagnetic materials are nowadays widely used as technological tools, especially for magnetic data storage. The modeling of small ferromagnets is based on the micromagnetic theory, an intriguing example of multiscale, nonconvex and nonlocal variational problems. One of the main challenges consists in understanding the pattern formation of the magnetization, in particular the domain walls. The aim of my talk is to present recent progress in the analysis of domain walls such as Bloch and Néel walls. I will present several results concerning their structure, their properties (stability, symmetry etc.) as well as the interaction energy of domain walls. The proof of these results is based on methods coming from geometric analysis and harmonic maps, elliptic regularity theory, variational methods and hyperbolic conservation laws.

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CONCENTRATION VERSUS OSCILLATION EFFECTS IN BRITTLE DAMAGE

Flaviana Iurlano
Sorbonne Université, France
Joint work with **Jean-François Babadjian and Filip Rindler**.

This work is concerned with an asymptotic analysis, in the sense of Γ-convergence, of a sequence of variational models of brittle damage in the context of linearized elasticity. The study is performed as the damaged zone concentrates into a set of zero Lebesgue measure and, at the same time and to the same order ε, the stiffness of the damaged material becomes small. Three main features make the analysis highly nontrivial: at ε fixed, minimizing sequences of each brittle damage model oscillate and develop microstructures; as $\varepsilon \to 0$, concentration of damage and worsening of the elastic properties are favoured; and the competition of these phenomena translates into a degeneration of the growth of the elastic energy, which passes from being quadratic (at ε fixed) to being linear (in the limit). Consequently, homogenization effects interact with singularity formation in a nontrivial way, which requires new methods of analysis. We explicitly identify the Γ-limit in two and three dimensions for isotropic Hooke tensors. In the limit, a surprising expression for the bulk density appears involving now a continuum damage variable. We further consider the regime where the divergence is square-integrable, which in the limit leads to a Tresca-type plasticity model.

DOMAIN PATTERN FORMATION IN FERROMAGNETIC SAMPLES

Hans Knüpfer
Universität Heidelberg, Germany

We investigate the optimal shape and patterns for magnetic domains from the perspective of Calculus of Variations. These patterns are driven mainly by the competition of local energies such as interfacial energies and anisotropy energy and the nonlocal magnetostatic interaction. In particular, we are interested in pattern formation in thin ferromagnetic films with perpendicular anistropy and in the phase transformation for non-trivial states. For the analysis, we do not use

any assumptions on the shape of the domain, rather the arguments are based on the derivation of suitable interpolation inequalities.

TOTAL VARIATION AND PHASE FIELD REGULARISATIONS OF AN INVERSE PROBLEM WITH QUASILINEAR MAGNETOSTATIC EQUATIONS

Kei Fong Lam
Chinese University of Hong Kong, Hong Kong
Joint work with **Irwin Yousept**.

We tackle the inverse problem of reconstructing a discontinuous coefficient in magnetostatic equations from measurements in a subdomain. This problem is motivated from identifying the location of magnetic materials (e.g. iron) in a bounded domain containing also non-magnetic materials (e.g. copper), and can be viewed as an idealised problem for non-invasive/non-destructive testing based on electromagnetic phenomena. The magnetic material produces a stronger response compared to the non-magnetic material in the presence of an applied current field, and the situation can be well-described by quasilinear H(curl) magnetostatic equations. As the inverse problem is likely to be ill-posed, we reformulate it into a constraint minimisation problem with perimeter penalisation. Existence of minimisers, stability with respect to data perturbation, and consistency as the penalisation parameter tends to zero are discussed. We then introduce a further phase field approximation of the minimisation problem and derive the first order necessary optimality conditions. Then, we investigate the sharp interface limit to demonstrate the phase field approximation is a meaningful method to solve the inverse problem.

ENERGY RELEASE RATE IN PLANAR ELASTICITY IN PRESENCE OF REGULAR CRACKS

Ilaria Lucardesi
Université de Lorraine, France
Joint work with **Stefano Almi and Giuliano Lazzaroni**.

In this talk, we first analyze the singular behavior of the displacement of a linearly elastic body in dimension 2 close to the tip of a C^∞ crack, extending the well-known results for straight fractures [4]. As conjectured by Griffith [3], the displacement behaves as the sum of an H^2-function and a linear combination of two singular functions, whose profile is similar to the square root of the distance from the tip. The coefficients of the linear combination are the so called *stress intensity factors*. Afterwards, we prove the differentiability of the elastic energy with respect to an infinitesimal fracture elongation and we compute the *energy release rate*, enlightening its dependence on the stress intensity factors [2]. In the last part of the talk we present the generalization to $C^{1,1}$ fractures and an application to crack evolution [1].

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EXISTENCE AND REGULARITY FOR THE NON-LINEAR KOITER SHELL INTERACTING WITH THE $3D$ INCOMPRESSIBLE FLUID

Boris Muha
University of Zagreb, Croatia
Joint work with **Sebastian Schwarzacher**.

We study the unsteady Navier Stokes equations in three dimensions interacting with a non-linear flexible shell of Koiter Type. The latter one constitutes a moving part of the boundary of the physical domain of the fluid. This leads to a coupled system of non-linear PDEs with the moving boundary. We study weak solution to the corresponding fluid-structure interaction (FSI) problem. We introduce new methods that allow to prove higher regularity estimates for the shell. Due to the improved regularity estimates it is then possible to extend the known existence theory of weak solutions to the FSI problem with non-linear Koiter shell. The regularity result holds for arbitrary weak solution under certain geometric condition on the deformation of the boundary.

DYNAMIC SCALING IN COAGULATION EQUATIONS

Barbara Niethammer
Universität Bonn, Germany

In 1916 Smoluchowski derived a mean-field model for mass aggregation in order to develop a mathematical theory for coagulation processes. Since Smoluchowski's groundbreaking work his model and variants of it including fragmentation terms have been used in a diverse range of applications such as aerosol physics, polymerization, population dynamics, or astrophysics.

In this talk I will give an overview on recent work, studying the long-time behaviour of such equations. A key question is that of dynamic scaling, that is whether solutions develop a universal self-similar form for large times. This issue is only understood for some exactly solvable cases, while in the general case most questions are still completely open. I will give

an overview of the main results in the past decades and explain why we believe that in general the scaling hypothesis is not true.

RECENT RESULTS ON ADDITIVE MANUFACTURING GRADED-MATERIAL DESIGN BASED ON PHASE-FIELD AND TOPOLOGY OPTIMIZATION

Elisabetta Rocca
University of Pavia, Italy
Joint work with **Ferdinando Auricchio, Elena Bonetti, Massimo Carraturo, Dietmar Hoemberg, and Alessandro Reali**.

A novel graded-material design for additive manufacturing based on phase-field and topology optimization is introduced by means of an additional phase-field variable in the classical single-material phase-field topology optimization algorithm. This new variable is used to grade the material properties in a continuous fashion. Different numerical examples are discussed and first order optimality conditions are obtained including possible stress constraints in the objective functional. From the presented results we can observe that the proposed algorithm adds additional freedom in the design, exploiting the higher flexibility coming from additive manufacturing technology.

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MATHEMATICAL MODELS OF INTERDIFFUSION WITH VEGARD RULE

Lucjan Sapa
AGH University of Science and Technology, Poland
Joint work with **Bogusław Bożek and Marek Danielewski**.

We study the diffusional transport in an s-component solid solution. Let $\Omega \subset \mathbb{R}^n$ be an open and bounded set with a piecewise smooth boundary $\partial\Omega$ and let $T > 0$ be fixed. Moreover, let Ω_i, D_i, j_i and c_{0i} mean the partial molar volumes, the diffusion coefficients, the evolution of a mass through $\partial\Omega$ and the initial concentrations. The unknowns are the concentrations of the components of a mixture c_i and the potential F of a drift velocity.

The local mass conservation law for fluxes with the Darken drift term and the Vegard rule lead to the parabolic-elliptic system of strongly coupled nonlinear differential equations

$$(1) \qquad \begin{cases} \partial_t c_i + \operatorname{div}\big(-D_i(c_1,...,c_s)\nabla c_i + c_i \nabla F\big) = 0 & \text{on} \quad [0,T]\times\Omega, \\ \triangle F = \operatorname{div}\big(\sum_{k=1}^{s} \Omega_k D_k(c_1,...,c_s)\nabla c_k\big) & \text{on} \quad [0,T]\times\Omega, \\ \int_\Omega F dx = 0 & \text{on} \quad [0,T], \end{cases}$$

with the nonlinear coupled initial-boundary conditions

$$(2) \qquad c_i(0,x) = c_{0i}(x) \quad \text{on} \quad \Omega,$$

$$(3) \qquad \begin{cases} -D_i(c_1,...,c_s)\frac{\partial c_i}{\partial \mathbf{n}} + c_i \frac{\partial F}{\partial \mathbf{n}} = j_i(t,x) & \text{on} \quad [0,T]\times\partial\Omega, \\ \frac{\partial F}{\partial \mathbf{n}} = \sum_{k=1}^{s} \Omega_k \big(D_k(c_1,...,c_s)\frac{\partial c_k}{\partial \mathbf{n}} + j_k(t,x)\big) & \text{on} \quad [0,T]\times\partial\Omega, \end{cases}$$

for $i = 1, ..., s$. This model was introduced in [3] and a some special case in [4]. In the one-dimensional case it can be transformed to the well-known model studied for example in [2, 5].

We will present theorems on existence, uniqueness and properties of weak solutions in the suitable Sobolev spaces. Moreover, finite implicit difference methods (FDM) and theorems concerned convergence and stability will be given. The agreement between the theoretical results, numerical simulations and experimental data will be shown.

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STABEL CONFIGURATIONS OF PRESTRAINED RODS

Francesco Solombrino
University of Naples Federico II, Italy
Joint work with **Marco Cicalese and Matthias Ruf**.

We study the stable configurations of a thin three-dimensional weakly prestrained rod subject to a terminal load as the thickness of the section vanishes. By Γ-convergence we derive a one-dimensional limit theory and show that isolated local minimizers of the limit model can be approached by local minimizers of the three-dimensional model. In the case of isotropic materials and for two-layers prestrained three-dimensional models the limit energy further simplifies to that of a Kirchhoff rod-model of an intrinsically curved beam. In this case we study the limit theory and investigate global and/or local stability of straight and helical configurations. We also show, by means of a bifurcation analysis, an exchange of stability between the straight configuration and a branch of local minimizers with so-called hemihelical shape, confirming experimental results.

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CHALLENGES IN DATA ASSIMILATION AND PREDICTION OF TROPICAL WEATHER AND CLIMATE

Samuel Stechmann
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Joint work with **Ying Li**.

Data assimilation is the process of combining (imperfect) observational data and (imperfect) model data in order to estimate the state of a complex system, such as the atmosphere. It is crucial to weather prediction as it provides the initial conditions for a forecast. In this talk, we describe the unique challenges that arise for tropical weather/climate, along with some recent results on estimating the intrinsic limits of predictability in the tropics using observational data.

ANALYTICAL AND NUMERICAL ASPECTS OF RATE-INDEPENDENT GRADIENT-REGULARIZED DAMAGE MODELS

Marita Thomas
WIAS, Germany

This presentation deals with techniques for the spatial and temporal discretization of models for rate-independent damage featuring a gradient regularization and a non-smooth constraint due to the unidirectionality of the damage process. A suitable notion of solution for the non-smooth process is introduced and its corresponding discrete version is studied by combining a time-discrete scheme with finite element discretizations of the domain. Results and challenges on the convergence of the discrete problems in the sense of evolutionary Gamma-convergence in dependence of the choice of the gradient term and the mesh properties are discussed. Directions towards adaptive strategies are pointed out. The presented results are based on collaborations

with S. Tornquist (WIAS), Ch. Kuhn and A. Schlüter (Kaiserslautern), S. Bartels and M. Milicevic (U Freiburg) and with M. Walloth and W. Wollner within the Priority Program SPP 1748 of the German Research Foundation.

INTRINSIC VIEWS IN ELASTO-PLASTICITY

Nicolas Van Goethem
University of Lisbon, Portugal
Joint work with **Samuel Amstutz**.

In this talk, I will present a new model of elasto-plasticity based on the elastic strain incompatibility. Existence results for a novel boundary value problem will be reported as well as asymptotic results. Indeed, this new system for incompatible elasticity generalizes the classical linearized elasticity system to which it converges as the incompatibility modulus converges to $-\infty$. I will also briefly introduce intrinsic views from a historical point of view.

A FAT BOUNDARY-TYPE METHOD FOR LOCALIZED NONHOMOGENEOUS MATERIAL PROBLEMS

Alex Viguerie
University of Pavia, Italy
Joint work with **Ferdinando Auricchio and Silvia Bertoluzza**.

Problems with localized nonhomogeneous material properties arise frequently in many applications and are a well-known source of difficulty in numerical simulations. In certain applications (including additive manufacturing), the physics of the problem may be considerably more complicated in relatively small portions of the domain, requiring a significantly finer local mesh

compared to elsewhere in the domain. This can make the use of a uniform mesh numerically unfeasible. While nonuniform meshes can be employed, they may be challenging to generate (particularly for regions with complex boundaries) and more difficult to precondition. To address the aforementioned challenges, we employ a technique related to the Fat boundary method [1] as a possible alternative. We analyze the proposed methodology from a mathematical point of view and validate our findings with a series of two-dimensional numerical tests.

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